Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

Eidgenössisches Departement des Innern EDI
Bundesamt für Meteorologie und Klimatologie MeteoSchweiz

Veröffentlichung MeteoSchweiz Nr. 84

Customer oriented warning systems

Jacques Ambühl

Herausgeber
Bundesamt für Meteorologie und Klimatologie, MeteoSchweiz, © 2010

MeteoSchweiz
Krähbühlstrasse 58
CH-8044 Zürich
T +41 44 256 91 11
www.meteoschweiz.ch

Weitere Standorte
CH-8058 Zürich-Flughafen
CH-6605 Locarno Monti
CH-1211 Genève 2
CH-1530 Payerne

Veröffentlichung MeteoSchweiz Nr. 84

ISSN: 1422-1381

Customer oriented warning systems

Jacques Ambühl

Bitte zitieren Sie diese Veröffentlichung folgendermassen
Ambühl, J: 2010, Customer oriented warning systems, *Veröffentlichungen der MeteoSchweiz*, **84**, 86 pp.

Herausgeber

MeteoSchweiz
Krähbühlstrasse 58
CH-8044 Zürich
T +41 44 256 91 11
www.meteoschweiz.ch

Weitere Standorte
CH-8058 Zürich-Flughafen
CH-6605 Locarno Monti
CH-1211 Genève 2
CH-1530 Payerne

Abstract

This study presents a theoretical analysis of the relationship between a warning organisation (issuer) and the users (addressees) of these warnings, in the field of weather forecasting. Warnings are based on probabilistic forecasts emanating from an Ensemble Prediction System, or from any system delivering a diagnostic expressed in term of probabilities. The addresse is characterized by the three key questions usually raised in risk analysis: What is the value at risk? How likely is a loss?, and what are the consequences, i.e. what happens under the course of actions available to him. Perspective of the addressee and the issuer are then linked together to develop a theory of optimal warning thresholds, enabling a qualitative translation of the issuer's performance into efficiency by the addressee.

.

Contents

1 Introduction

A theoretical analysis of the relationship between a warning organisation and the users of the warnings issued by this organisation is presented. The warning organisation is usually a weather service. Users might be for example farmers, hydrological services, airport authorities or management boards of power plants. The warning organisation will thereafter be referred to as the issuer, the user of the warnings as the addressee. They will be referred to as "the actors" when considered together. The methodology is inspired by numerous mathematical developments that occurred during the last decades in finance engineering, aimed at providing investors with optimal decisional schemes.

The programme is realised by establishing a clear distinction between both actors.

The addressee is characterised by a risk profile and an economic profile. His risk profile is composed by the risks his business is confronted with (e.g. lost of a harvest in case of precipitation), as well as by the climatology he is exposed to (e.g. the probability of occurrence of extreme precipitation). Both risk and climatology are then merged in the relative operating characteristics of the addressee. His economic profile is inspired by the classical work of D. S. Richardson, [1]. It is defined by the triad of the loss resulting in an unpredictable event for which neither mitigating nor protective actions were taken, the cost induced by the protective actions taken in case of occurrence as well as in case of non occurrence of an event, and finally the residual cost occurring in the case of a well predicted event for which protective measures were (adequately) taken. These various risks and economic factors being fairly entangled, an objective of this work is to provide, at least at theoretical level, some clarity in that matter.

The issuer is expected to base his warning on probabilistic forecasts emanating from an Ensemble Prediction System (EPS), or on any system delivering a diagnostic expressed in term of probabilities. The performance of the issuer is then characterized by a dedicated Relative Operating Characteristic (ROC).

The adequate tuning of the warning system enables the maximization of the addressee's benefit through the determination of optimal warning thresholds. The dualistic approach proposed provides a quantitative relationship between the performance objectives having to be reached by the issuer and the monetary outcome expected by the addressee. Enabling the assessment of the issuer's performance impact onto the addressee's efficiency, this approach satisfies the requirements formulated in the realm of New Public

Management projects undertaken in several weather services. It allows the settlement of a service level agreement between both actors.

Furthermore, an accurate knowledge of the profile of the addressee facilitates the elaboration of specifically customer-tailored products, in the same vein as structured products modified the financial realm decades ago[1].

2 Outline of the problem

At this point, it is worth sketching the differences between a warning system, an insurance contract and a weather derivative product. Weather derivatives are conditional contracts established between a finance institute and a stakeholder whose business is weather dependent. Weather derivative payouts solely depend on the outcome of the weather, regardless of how it affects the profit of the holder. On the contrary, the holder of an insurance contract has to prove that he has suffered a financial loss induced by adverse weather that actually occurred in order to be compensated. Warning systems are aimed at guiding stakeholders, i.e. addressees, in the triggering of protective or mitigating actions in the prospect of adverse weather events.

Conclusively, insurance contracts are conceived in order to cover damages on real estates and objects, weather derivative to protect financial flows. In a first approximation, warning systems and insurance contracts are designed to cope with extreme weather events, weather derivative are to cover financial risks induced by rather normal weather fluctuations, [2]. The boundaries between these businesses being fuzzy, the issue will be briefly addressed.

Figure 1 sketches a schematic relationship between these various domains.

[1]Various other systems could benefit from this study, including road and aviation authorities, civil protection agencies, tsunami and hurricane warning services, etc.

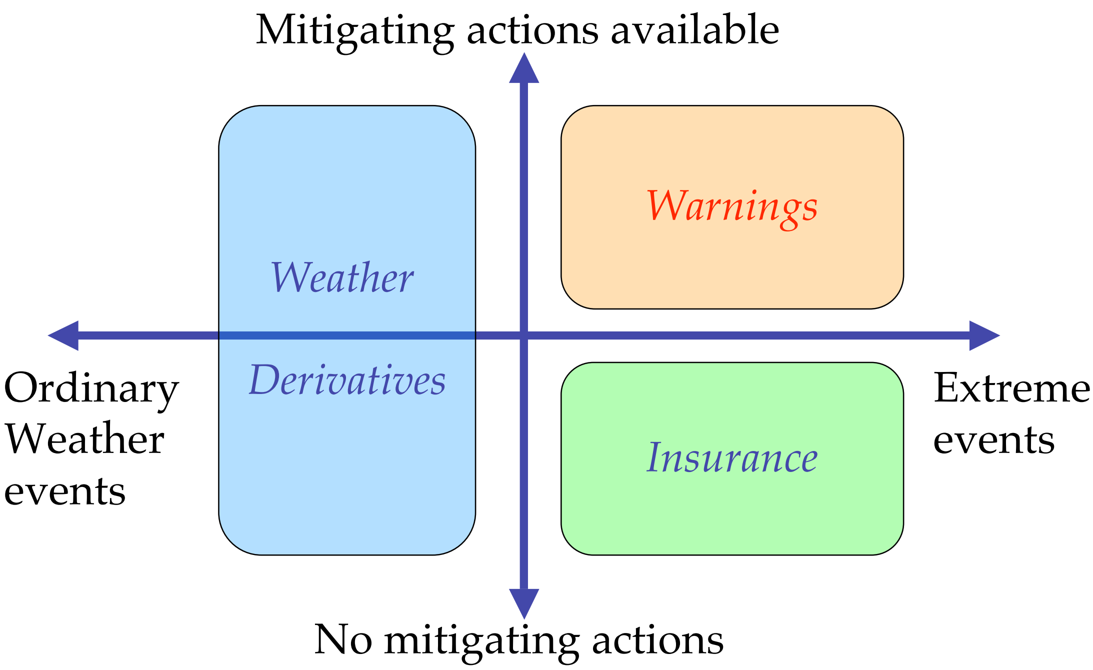

Figure 1: *Schematic representation of the four possible combinations of common or extreme weather events, for which mitigating actions may or may not be available. Sketch of three corresponding business fields. (Evenly could in practice warnings be issued for benign weathers events too, as for example when agriculturists see their harvests threatened by moderate rainfalls).*

3 Scope and Aim

This essay is devoted to the study of warning systems tuned in order to cope with the events sketched on the upper right corner of Figure 1. The approach is dualistic. It consists in establishing and adamantly maintaining a clear demarcation line between both actors, addressee and issuer, and then in studying the interaction occurring between them.

The methodology is based on theoretical considerations whose consequences are verified against numerical simulations. To this end, the conceptual model of a warning system, tuned in order to operate optimally under prevailing climatic conditions and addressee's requirements, is introduced.

The document is organised in two layers. Leading concepts and related discussions are introduced in the body of the text, Sections 4 to 11. Ancillary developments are provided in the Appendix, Section 12. The self-consistency of the core is as far as possible warranted, thus making references to the Appendix optional.

Acknowledgment

This study sumarises the result of personal reflections conducted on private basis in 2006 and 2008. Nans Addor, Martina Amstutz, Tamara Comment, Thomas Egli, Graham Knapp, Christoph Rheinberger, Mathias Rotach, Philippe Steiner, and Christophe Voisard deserve special thanks for their help and contributions to the reviewing of this document.

4 Brief historic and basic definitions

During the last decades, various forecasting scores have been introduced in order to assess the quality and the efficiency of weather forecasting and warning systems. The widespread of probabilistic forecasts based on Ensemble Prediction Systems has favoured the diffusion of performance indicators based on such systems, [3]. The hit rate, the false alarm rate and false alarm ratio have emerged as the key parameters used for this purpose. In 2003, ECMWF published the "Recommendations on the verification of local weather forecasts", [4]. Having almost mandatory status among ECMWF member states, they prescribe the use of the aforementioned scores. Accordingly, the present essay is written in conformity with the typographical standards provided by these recommendations. Those scores are succinctly defined below.

The hit rate[2], expresses the ratio between the number of correctly warned events and the total number of events. Measuring the overall success of the warning system, it is of paramount importance for both actors.

On the other side, the subtle difference between the false alarm rate and the false alarm ratio should not be underestimated. The false alarm rate measures the frequency at which the addressee's business, instead of running smoothly, is unnecessarily impeded by protective actions taken under fair weather conditions. The false alarm ratio, expressed as the ratio between the number of mistakenly issued warnings and the total number of issued warnings, provides a measure of the quality of the service provided by the issuer.

Although the hit rate is relevant to both actors, the false alarm rate is a measure of the efficiency of a warning system, as perceived by the addressee, and the false alarm ratio a measure of the performance of that warning system, in issuer's hand. This fact is illustrated through a brief numerical example presented in the Appendix, Section 12.1. Conclusively, the false alarm rate happens to be an addressee's issue, the false alarm ratio an issuer's concern. The dual analysis of both profiles, hereafter presented, is a direct consequence of that observation.

5 Addressee's profile

The object of this Section consists in elaborating the profile of a so-called rational addressee. It is firstly based on the accounting of weather related hazards he is facing to, then on the evaluation of the financial burden thereby

[2]Also named Probability of Detection.

induced. Rather than trying to compute a solid monetary outcome having to be paid, this Section is aimed at identifying and explaining the relationship between the statistical - climatological information available and the financial constraints the addressee is confronted with.

5.1 Addressee's risk profile

5.1.1 Probabilistic formulation of the risk

Instead of working with *a posteriori* computed frequencies, it is worthwhile to consider probabilistic functions representing the expected distributions of the events taken into consideration. Two of them are introduced in the present setting: on the one hand a representation of the climatology the addressee is exposed to, on the other hand the frequency of the disasters he is confronted with. Both are expressed in terms of one weather parameter, e.g. temperature or gale intensity or cumulated precipitation fallen within a period of time. This period of time, which can last for a few hours, a day, a week, will be referred to as Δ.

The frequency of the disasters might be expressed as for example in terms of the relative increase of medical emergencies occurring during a heat wave or in terms of the frequency of interventions of emergency crews in the case of a storm, all of them being expected to occur during Δ.
The span of the meteorological parameter has to be specified, for example between 0 and 200 km/h for gales (at least in Switzerland), or between 0 and 400 mm precipitation, within Δ. Those inferior and superior bounds are referred as *0* and B in the sequel, with the meteorological parameter being expressed in arbitrary ***units***.

More specifically, the two following probabilistic distributions are now introduced:

The climate profile Figure 2, probability of occurrence, during the period Δ, of a weather event W, the intensity of which lies between q and $q + dq$:

$$C_{(q)}dq = \Pr[W \in [q, q + dq] \; ; \; \Delta] \; ; \; q \in [0, B] \quad (1)$$

The disaster profile Figure 3: probability of occurrence during the period Δ of a disaster D induced by a weather event W the intensity of which lies between q and $q + dq$:

$$\mathcal{E}_{(q)}dq = \Pr[D \in \Delta; \; W \in [q, q + dq]]; \; q \in [0, B] \quad (2)$$

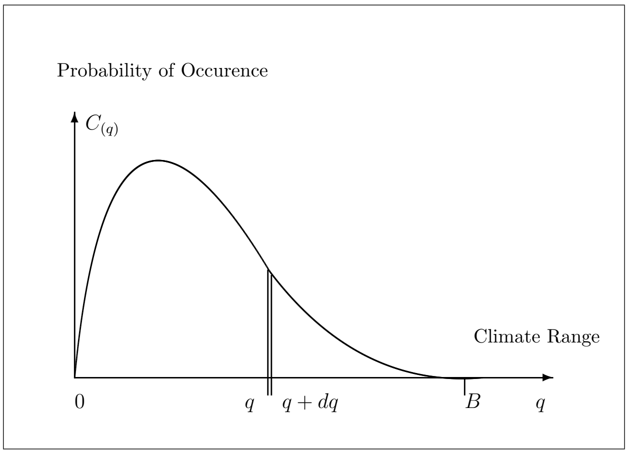

Figure 2: *Climatic profile: Probabilistic distribution of weather events.*

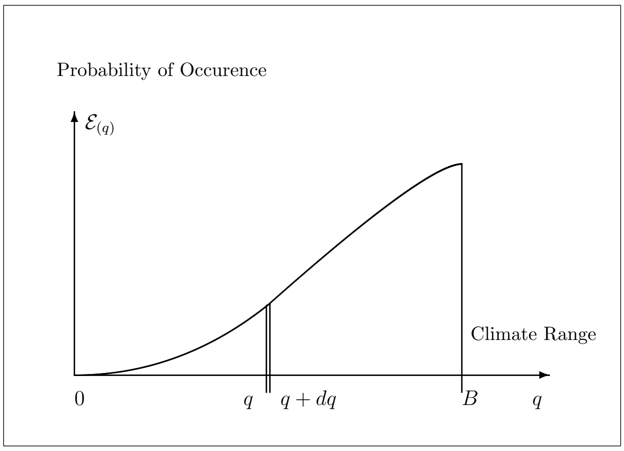

Figure 3: *Disaster profile: Probabilistic distribution of induced disasters*

Both distributions (1) and (2) are presented again in Figure 4, this time with a ***meteorological threshold*** Q sketched as the downward pointing vertical arrow. One notices that most meteorological events occur at low intensity and are figured by the belly of the C distribution. Extreme events are represented by the right tail of the C distribution.

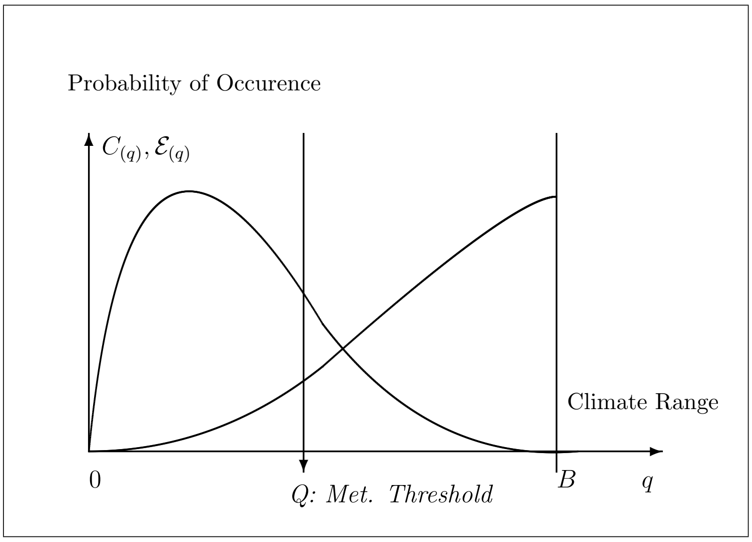

Figure 4: *Climate versus disaster profiles. Basis for the construction of the relative operating characteristic.*

5.1.2 Climate burden and exposure

The climate burden is defined as the financial impact induced by adverse weather events onto the addressee's business if he disregards mitigating or protective actions. Let us formalize this notion and consider $\int_0^B C_{(q)}\mathcal{E}_{(q)}dq$, measuring the overall probability of occurrence of weather induced disasters. Introducing the cost $l_{(q)}$ caused by one disastrous event occurring at weather intensity q, expressed in an arbitrary monetary unit, the climate burden is given by $\int_0^B C_{(q)}\mathcal{E}_{(q)}l_{(q)}dq$. This expression integrates over the domain $[0, B]$ the frequency of occurrence $C(q)\mathcal{E}_{(q)}$ of weather induced disasters at intensity q, multiplied by the loss $l(q)$ engendered at that intensity.

Let us now assume that, for the addressee - or at least for his wallet - 10 disasters induced by one weather event and costing 10 monetary units each are equivalent to one disaster induced by that event and costing 100 monetary units. This allows us to introduce a reference disaster loss L and a dimensionless equivalency factor $m_{(Q)}$ defined as the ***loss multiplier***. Taking those elements into account, the cost $l_{(q)}$ caused by one disastrous event occurring at weather intensity q: $l_{(Q)} = L\, m_{(Q)}$ is well defined. Transported into the previous integral, one has: $L \int_0^B C_{(q)} \mathcal{E}_{(q)}\, m_{(q)} dq$. Now, the product $\mathcal{E}_{(q)}\, m_{(q)}$ defines a new profile, expressing through an increased probability of occurrence the impact of the equivalence factor $m_{(Q)}$ onto the disaster profile. Let it be defined as $E_{(q)} = \mathcal{E}_{(q)}\, m_{(q)}$. The ***climate burden***, taking variable disaster losses into account, is then given by:

$$L\,\Omega = L \int_0^B C_{(q)}\, E_{(q)}\, dq \tag{3}$$

Variable disaster costs have simply been encapsulated into the newly defined and dimensionless $E_{(q)}$ probability:

$$E_{(q)} dq = \Pr[\mathcal{D} \in \Delta;\ W \in [q, q + dq]];\ q \in [0, B] \tag{4}$$

where $\mathcal{D}$ defines the fictitious event taking into account the loss multiplier. To distinguish $E_{(q)}$ from the disaster profile, it will be defined as the ***addressee's exposure*** in the sequel.

The construction of disaster profiles and loss multipliers is usually an arduous undertaking. Experts might be gathered in panels to outline salient features of such profiles. Valuable insights are also provided by conversant customers and business partners. The model chosen in the present work is described in the Appendix, Section 12.4.1. The result is presented in Figure 5.

5.2 Relative Operating Characteristic

As the probability of induced disasters grows for increasingly extreme weather events (E distribution), the addressee is required to determine an optimal meteorological threshold Q at and beyond which mitigating actions should be undertaken. The introduction of hit and false alarm rates, as well as the corresponding relative operating characteristic, deserves this purpose.

Hit rate and false alarm rate are classically introduced as arithmetic ratios computed over accumulated cases (Appendix, Section 12.1). However, using the definitions introduced so far and following the development given in the Appendix, Section 12.2, hit and the false alarm rates might constructed as functions depending on the probabilistic distributions of climatic and disaster profiles. They are defined as:

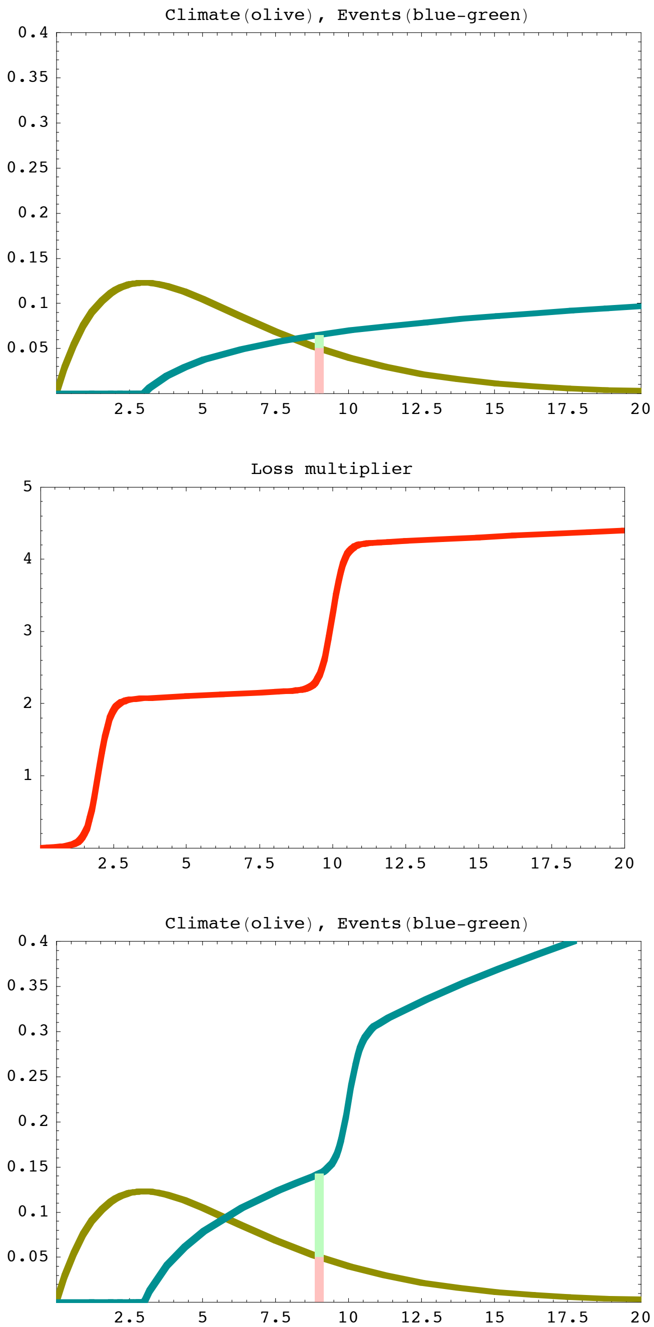

Figure 5: *Impact of the loss multiplier. Upper panel: climate and disaster profiles, as presented on Figure 4. Middle panel: possible profile of a loss multiplier. Losses are negligible below 3 units and double when weather events reach 10 units intensity. Lower panel: resulting addressee's exposure. Pink and light green bars sketched at meteorological intensity* $Q = 9$ *units express the increase in exposure at that meteorological threshold.*

Hit Rate: expresses the ratio between the probability of disasters occurring in the domain $[Q, B]$, for which mitigating actions are undertaken and the overall probability of occurrence of weather induced disasters:

$$H_{(Q)} = \frac{1}{\Omega} \int_{Q}^{B} E_{(q)} C_{(q)} dq \tag{5}$$

False Alarm Rate: expresses the ratio between the probability of non-occurrence of disasters in the domain $[Q, B]$ for which mitigating actions are inadequately undertaken and the overall probability of occurrence of weather conditions not triggering disasters:

$$F_{(Q)} = \frac{1}{1-\Omega} \int_{Q}^{B} (1 - E_{(q)}) C_{(q)} dq \tag{6}$$

These expressions, taking their values in the interval $[0, 1]$, are functions of the meteorological threshold Q.

Having those two parameters at hand, the addressee's ***relative operating characteristic - ROC -*** can be established. Presented in Figure 6, it is illustrated by the curve whose abscissa is expressed in terms of false alarm rate, whose ordinate is expressed in terms of hit rate and that is parametrized in terms of meteorological thresholds Q. The parameter runs from low meteorological thresholds (corresponding to high hit rates and false alarm rates), located at the top right corner of the diagram, to high meteorological thresholds (corresponding to low hit rates and false alarm rates) at its bottom left corner. In between, the ROC-curve moves into the vicinity of the top left corner, where the hit rate is rather high, the false alarm rate rather low.

A sound addressee would choose a threshold in this area. In first approximation, addresses opting for low thresholds could be deemed ***risk adverse***. They are inclined to trigger mitigating actions even by faint evidence of an incoming disaster. Consequently, they pay with a substantial increment in false alarm rate a marginal improvement in hit rate. Correspondingly, addressees who decide to operate at high thresholds (with low false alarm rate and hit rate) are ***risk friendly***. They are willing to take mitigating actions only when the evidence for an incoming disaster is high. They also require a substantial increase in hit rate in order to endorse even a tiny increment in false alarm rate.

Two remarks: A more formal definition of risk awareness implies considerations related to the (economic, societal or even moral) impact of a decision. Furthermore, the kind of awareness presented here focuses on the

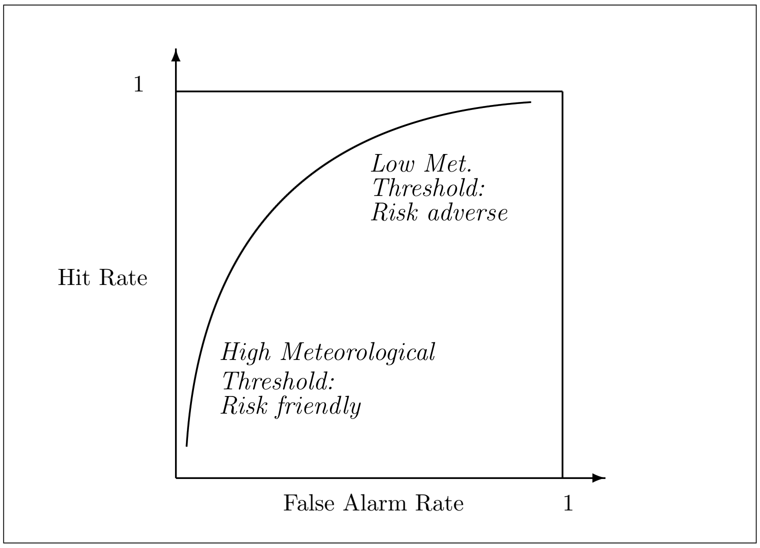

Figure 6: *Relative operation characteristic and addressee's risk awareness*

impactive risk on the addressee's business. In a dual interpretation, the issuer's point of view will be considered. In that dual perspective, an issuer giving out warnings at low thresholds, thus possibly warning too frequently, would happen to behave in a risk friendly manner. Both points will be further refined in Sections 5.4, 6.5 and 8.3.

The concepts discussed so far are presented in Figure 7. The relationship between climate profile and addressee's exposure is repeated on the top panel. The addressee experiences no damage for events occurring at intensity lower than or equal to 3 units. The corresponding relative operating characteristic is given on the bottom panel. The meteorological threshold is marked by the green dot. By very low thresholds, below 3 units, mitigating actions being always taken, the hit rate equals 1 and the false alarm ratio is high. Both converge towards zero by high thresholds.

5.3 Concluding remark

The relative operating characteristic enables the fusion of the information related to the occurrence of climatic events and their consequences onto the addressee's business. It is to be interpreted as the addressee's ***exposure profile***.

The meteorological threshold happens so far to be the governing parameter through which the addressee controls the amount of risk he is willing to cope with (Figures 6 and 7). A methodology has therefore to be introduced, yielding the computation of an optimal meteorological threshold for his business. Considerations related to his economic profile, presented in following Section 5.3, solve this indeterminacy.

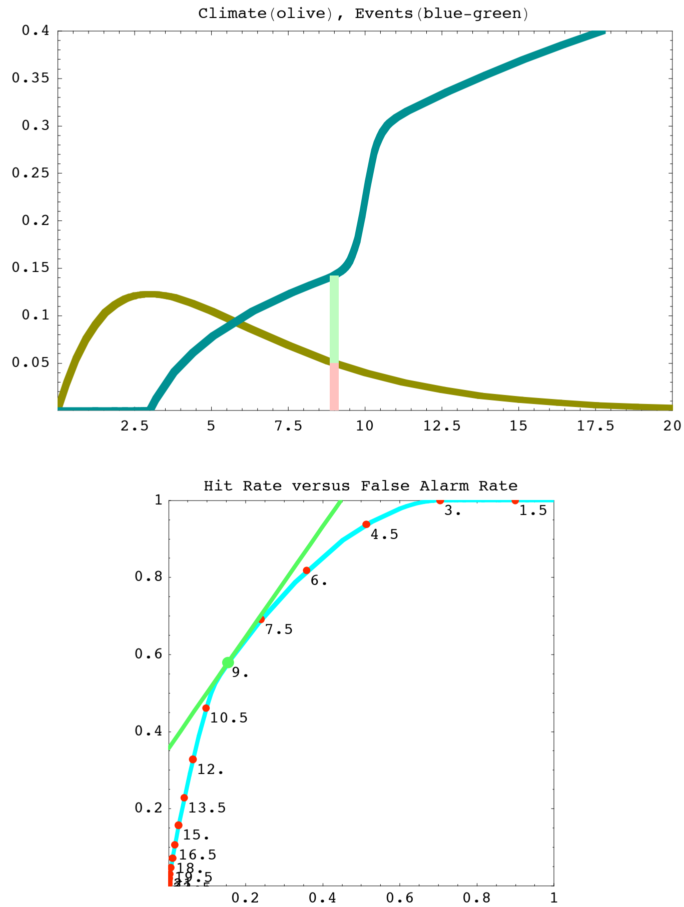

Figure 7: *Top panel: example of the relationship between climate profile and addressee's exposure with a meteorological threshold set again at* $Q = 9$ *units. Abscissa: climate range; ordinate: probability of occurrence. Bottom panel: corresponding relative operating characteristic. Abscissa: false alarm rate; ordinate: hit rate. The straight line tangent to the disaster profile at 9 units is related to the addressee's economic profile that will be introduced in Section 5.3.*

5.4 Addressee's economic profile

The profile is expressed following the definitions and a slight generalisation of the theory proposed by Richardson, [1]. This theory provides a measure of the economic impact induced by the occurrence of four possible situations described in the following contingency Table 1. The elements are expressed in monetary units: L represents the loss induced by a disaster for which no mitigating actions were taken. C represents the costs induced by such mitigating actions. They are due in case of occurrence of a correctly warned event, as well as in case of a mistakenly warned non event. λ represents the residual costs remaining in the case of a correctly warned event. It is assumed that $\lambda \ll L$ and $C < L$.

	event did not occur	event did occur
mitigating actions taken	C	$C + \lambda$
no mitigating actions taken	0	L

Table 1.

Although not present in the original paper by Richardson, the λ parameter is easily introduced and happens to provide a valuable generalisation.

The average costs M the addressee is faced to during a period long enough to be of climatological relevance can now be evaluated. They are given by:

$$M = \frac{1}{a+b+c+d}\,[bL + Cc + (C+\lambda)d] \tag{7}$$

with a, b, c and d defined in the Appendix, Section 12.1. Following Richardson, M can be expressed in terms of frequency of occurrence of the event (Ω), hit rate (H) and false alarm rate (F):

$$M_R(H,F) = L\,[F\,\Gamma\,(1-\Omega) + H\,\Omega\,(\Gamma + \Lambda - 1) + \Omega] \tag{8}$$

The derivation of this expression is provided in Appendix, Section 12.5.1. Besides the ***cost-loss ratio*** $\Gamma = \frac{C}{L}$, the pivotal parameter in this study, the parameter $\Lambda = \frac{\lambda}{L}$, baptized ***residual-loss ratio***, is introduced as well. Expression (8), whose Graph is presented in Figure 8, is the exact equivalent of equation (7), however formulated in terms of hit rate and false alarm rate.

The Richardson's economic function, being expressed in hit rate and false alarm rate, can be projected onto the ROC-frame $\{[0,1] \times [0,1]\}$. Furthermore, being linear in hit rate as well as in false alarm rate, its isolines, computed for a constant monetary values $M_\$$ by $M_R(H,F) = M_\$$, are straight lines in the ROC-frame. Sketched in Figure 9, they are called ***iso-costs*** in the following: as $[H, F]$ moves along an iso-cost, no change occurs to the financial burden the addressee is faced to. From his perspective, iso-costs are perceived as lines of equal sensitivity or, indeed, as iso-tolerance lines.

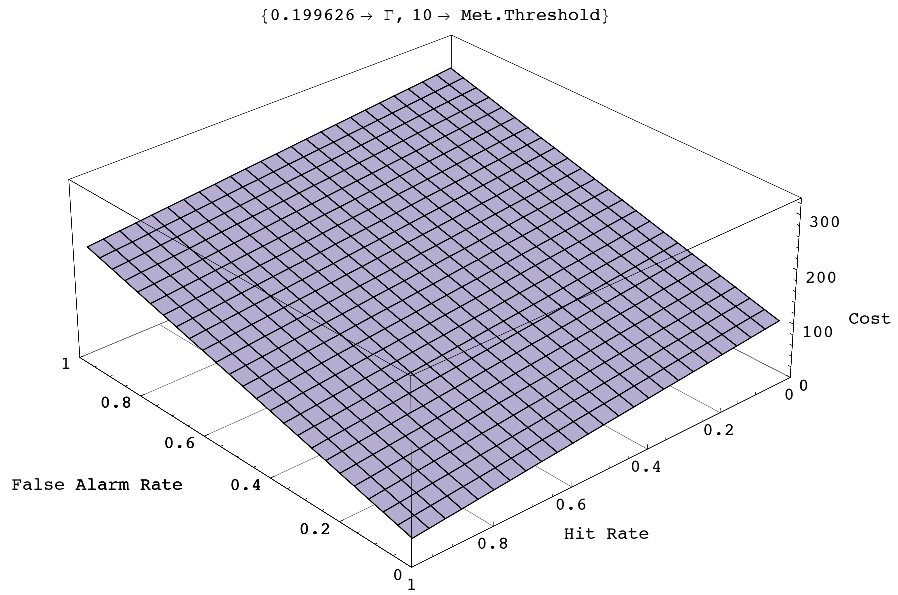

Figure 8: *Richardson's economic function. The hit rate runs along the right edge of the cube, the false alarm rate along the left edge. The cost-loss ratio is arbitrarily fixed at* $\Gamma = 0.1996$. *The function being linear, its graph is a plane in the space [false alarm rate* $\times$ *hit rate* $\times$ *Costs].*

A simple analysis of the structure of the Richardson expression (8) shows that the slope of the iso-costs, expressed as $\frac{\Delta H}{\Delta F}$, is governed by the value of the cost-loss ratio Γ (Figure 9). Thus, expressed in the ROC-frame, the iso-costs and their slopes represent the synthetic parametrization of the economic profile of the addressee, with the cost-loss ratio acting as the governing parameter.

Addressees frequently ignore the actual value of their own cost-loss ratio. Few examples (given in [3]) and one documented statistic for heat wave induced mortality in Switzerland, [5], are provided hereafter for illustration (with Λ parameter set to zero):

- $\Gamma = 0.02 - 0.05$ for orchardists (Murphy 1977)
- $\Gamma = 0.01 - 0.12$ for fuel-loading of aircraft (Leigh 1995)
- $\Gamma = 0.125$ for winter road gritting (Thornes and Stephensen 2001)

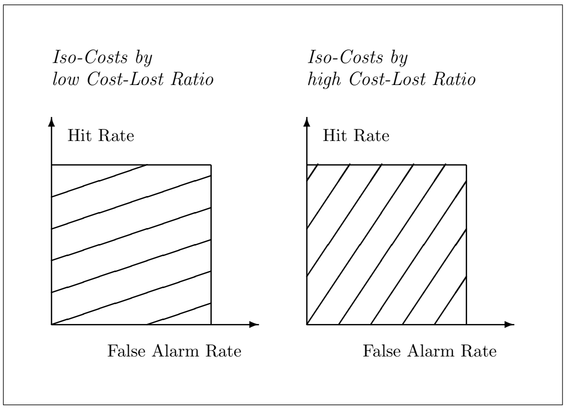

Figure 9: *Slope of the iso-costs in relation to cost-loss ratios. Costs are minimum at the top left corner of the diagram, maximum at the bottom right corner. iso-costs are interpreted as the addressee's economic profile in the following.*

- $\Gamma = 0.03 - 0.18$, Mortality and temperature in Switzerland 1990 - 2003 (Herren, personal communication and [5])

It should be deemed cynical to interpret mortality in terms of cost-loss ratio. Indeed, the relationship between the latter and the disaster profile will be demonstrated below in Section 5.3.

Up to now we have seen that the meteorological threshold is the parameter through which the addressee controls the amount of risk he is willing to cope with. It appears here that the cost-loss ratio plays a comparable role, this time emanating from the economic perspective. Although this interplay will constrain the addressee's freedom, it will clarify the risk-versus-reward dilemma he is faced with.

5.5 Rational addressee

The key elements relevant for the addressee when determining an optimal meteorological threshold Q are now at our disposal. His exposure profile, considered first, accomplishes the fusion of the information emanating from both climate profile and exposure curve, as defined by expressions (1) and (4). It is conveyed in terms of relative operating characteristic through the hit rate and the false alarm rate, equations (5) and (6). On the other side, the Richardson model of costs and losses, equation and Figure 8, describes his economic profile.

Both profiles being expressed in terms of hit rate and false alarm rate, they can be transported onto the ROC-frame, as presented in Figure 10 for two values of the cost-loss ratio. Considering that the addressee seeks his minimal financial burden in the long term, he will choose on the ROC the meteorological threshold providing the minimum value of the economic Richardson's function.

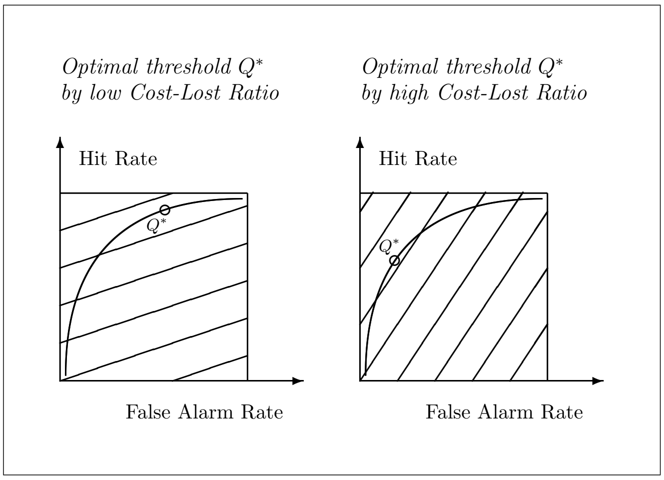

Figure 10: *Fusion of the exposure profile (ROC) and the economic profile (iso-costs) of the addressee. The minimum of the addressee's financial burden is reached at the tangency point of the ROC with the iso-costs. This defines the value optimal meteorological threshold (Q^* - circles).*

Noticing that the iso-costs are straight and the ROC concave, it appears that the optimal meteorological threshold is naturally located at the point of tangency of the iso-costs with the ROC (Figure 10). Thanks to the linearity of the iso-costs and the concavity of the ROC, it is the unique point of the ROC where the economic function reaches its minimum. According to the characterisation of risk awareness presented earlier, one immediately notices in Figure 10 that the addressee adopts a fairly risk adverse strategy when his cost-loss ratio is low, reciprocally a comparatively more risk friendly strategy when it is high.

As already mentioned, discussions with addresses make clear that they frequently ignore their cost-loss ratio, as well as the existence of a possible relationship with the meteorological threshold at which they might be warned. Indeed, they are faced with the following alternative:

The addressee knows his cost-loss ratio Γ: In this case, the slope of the iso-costs being given, seeking a minimum of his economic burden, he is bound to choose the meteorological threshold Q^* at that point on the ROC where tangency with the iso-costs occurs.

The addressee has fixed a meteorological threshold Q^*: Considering his choice as being economically optimal, he has implicitly assumed that the slope of the iso-costs are parallel to the tangent to the ROC at that chosen meteorological threshold. As a matter of fact, he has defined his cost-loss ratio Γ in the same fallen swoop.

Expecting a rational behaviour from the addressee, it is tempting to use the information available in order to formalise the issue. This information is threefold: it emanates 1) from the risk profile, provided by the ROC, 2) from the economic profile, as described by the iso-costs, and 3) from the requirement for optimality, expressed as the tangency requirement between iso-costs and ROC. According to the definitions established so far, this formal relationship is to be expressed in terms of meteorological threshold on the one side, in terms of cost-loss ratio on the other side. The connection between both sides is provided by the tangency requirement. It expresses the best addressee's trade-off between the variation of hit rate and the variation of false alarm rate. Those ratios having to be equal at the tangency point on the ROC, one has, as shown in Figure 10:

$$\frac{\partial_Q H}{\partial_Q F}|_{Riskprofile} = \frac{\partial H}{\partial F}|_{Economicprofile}.$$

This geometrical requirement defines a simple partial differential equation whose solution, presented in Appendix, Section 12.3, provides the expected relationship between the cost-loss ratio Γ, the residual loss ratio λ and the meteorological threshold:

$$E_{(Q)} = \frac{\Gamma}{1 - \Lambda} \tag{9}$$

This expression is of startling simplicity. It expresses the proportionality between the addressee's exposure $E_{(Q)}$ at a threshold Q and the cost-loss ratio, and equality between both if the residual-loss ratio, given by Λ, is set to zero. It is worth noticing that the Ω factor, representing the climate component, vanishes in the course of the derivation. Conclusively,

The addressee is qualified as rational if the relationship between his meteorological threshold Q^* and his cost-loss ratio Γ expresses the subsequent geometrical construction (Figure 11) and therefore satisfies equation (9) :

The notions of risk awareness and rationality are independent. According to his economic profile, an addressee can be rational **and** risk adverse, as well as rational **and** risk friendly or even rational **and** risk neutral. An improved, less intuitive definition of the risk awareness can now be proposed: An addressee will be considered risk adverse if it appears that, for him, the ratio $\frac{\Delta H}{\Delta F}$ is smaller than 1, respectively risk friendly if this ratio is larger than 1. Expressing the ratio of the variation of two dimensionless quantities, risk awareness is itself a purely differential concept given without monetary value of any kind. It must be furthermore emphasised that the requirement for $\frac{\Delta H}{\Delta F} \approx 1$ does not define rationality, but merely expresses the notion of risk neutrality. Reference [6] provides an outstanding introduction in the theory of risk awareness applied to financial engineering.

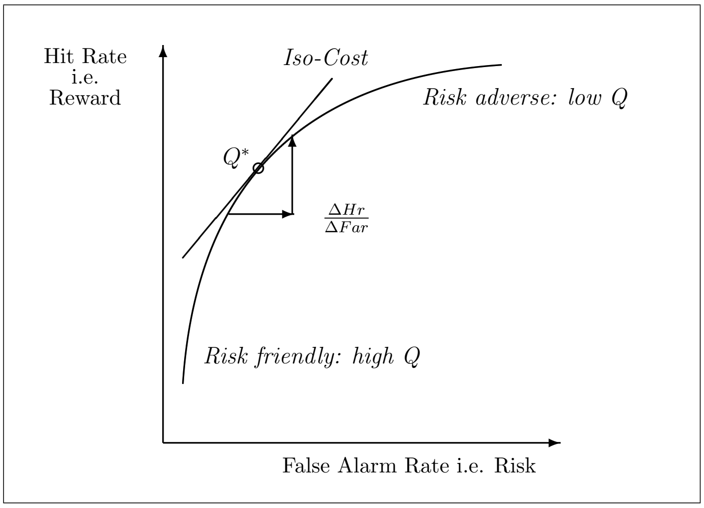

Figure 11: *Zoom on the tangency point between iso-cost and ROC for a* ***rational addressee:*** *being located at the point on the ROC where the Richardson function takes its minimum, the optimal meteorological threshold Q^* expresses the best possible $\frac{\Delta H}{\Delta F}$ ratio for the addressee, according to his economic profile.*

5.6 Summarising example

As a conclusion of this Section, two addressees are presented in Figure 12. Both are characterised by the same exposure, figured in the top row. Their economic profiles, however, differ (middle row). The left addressee, having a low cost-loss ratio, operates at low meteorological threshold (6 units). Conversely, the right addressee operates at higher cost-loss ratio and meteorological threshold (12 units). Whilst the former is fairly risk adverse and the latter definitely risk friendly, both behave rationally.

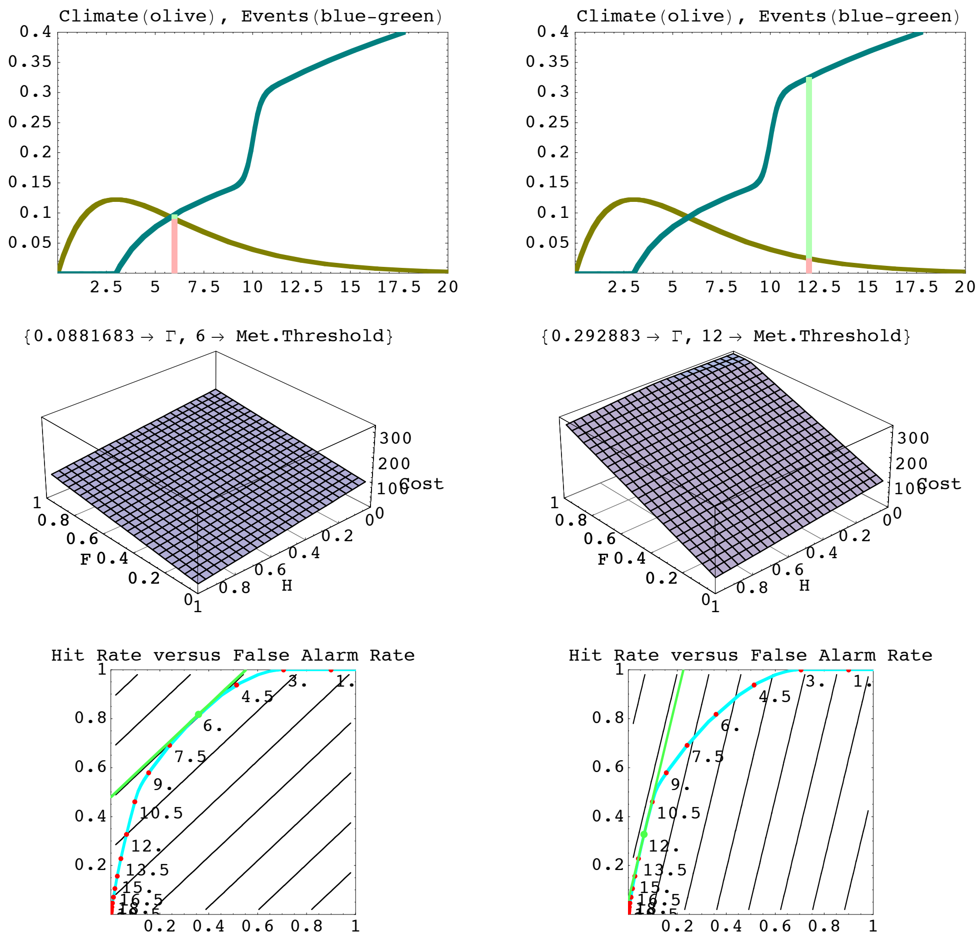

Figure 12: *Comprehensive representation of the relationship between meteorological thresholds and cost-loss ratios for two rational addressees. Left column: meteorological threshold = 6 units; cost-loss ratio = 0.088. Right column: meteorological threshold = 12 units; cost-loss ratio = 0.029. The addressee is fairly risk adverse on the left column, definitely risk friendly on the right one.*

6 Issuer's profile

Having discussed the addressee's characteristics so far, we will now focus our attention on the issuer, with meteorological services in mind. Considering that meteorological warnings are increasingly based on probabilistic forecasts, possibly emanating from Ensemble Prediction Systems, such a system will be simulated first. Later, the addressee's profile described in the previous Section will be injected onto the issuer's performance chart, expressed as ROC of hit rate and false alarm ratio. The synthesis of both actor's perspectives will then be presented and discussed in Section 7.

6.1 Warning system based on a simulated Ensemble Prediction System

The kernel of the work is the stochastic generator of an Ensemble Prediction System (EPS) that delivers simulated probabilistic forecasts. The methodology applied, consisting in building a sequence of random generators eventually producing the required probabilistic items, is presented in the Appendix, Section 12.4.2.

Characteristics of the simulated probabilistic forecasts and the corresponding weather events are presented in Figure 13. The climatic range in which simulated weather events occur is represented by the abscissa, spanning from 0 to B. The probability of their occurrence is expressed on the ordinate. The downward pointing thin arrow determines a meteorological threshold Q. A probabilistic forecast for the next verifying time is sketched as the (nearly Gaussian) distribution curve[3]. The corresponding weather event of intensity $\tilde{Q}$ is represented as the downward pointing thick arrow. Weather events as well as their corresponding forecasts are simulated in accordance with the probabilistic distribution of weather events given in Figure 2. This ensures that the addressee and the simulated forecasting model are subjected to the same climatology, a prerequisite for the envisaged confrontation of both actor's profiles.

The area below the probability distribution located at the right of the meteorological threshold Q, given by $\displaystyle\int_Q^B P(q)dq$, expresses the probability of the event "Weather occurs with an intensity equal or greater than Q", as forecasted by the simulated Ensemble Prediction System. (It is assumed that $\int_0^B P(q)dq = 1$). Having these elements at hand, it suffices now to introduce besides the meteorological threshold Q a ***probability threshold***

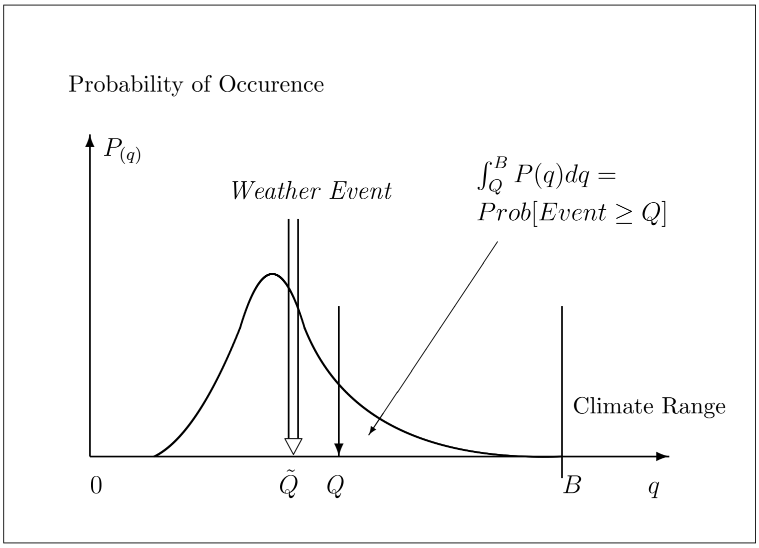

Figure 13: *Decision based on a probabilistic forecast. Q is the meteorological threshold. The area below the probability distribution located at the right of the meteorological threshold Q expresses the probability of the event "Weather occurs with an intensity equal or greater than Q", as forecasted by the Ensemble Prediction System.*

P to define a simulated warning system.

All elements required to design the issuer's contingency table, *defined for one meteorological threshold Q and one probability threshold P*, are now at our disposal. Based on a table $< event, forecast >$ produced by the EPS-simulator (Appendix, Section 12.4.2) and making use of the decision scheme given hereafter, they enable the construction of an arithmetic contingency table having the same structure as those tables presented in the Appendix, Section 12.1.

It must be stressed that this valuation can be performed on any probabilistic forecast. Indeed, a vast array of methods have been designed since the beginning of scientific meteorology in order to issue such forecasts. Three of them, deriving probabilistic short range forecasts from a deterministic high resolution model, or from radar information, References [7], [8] and [9] are mentioned here as recent examples.

6.2 Decision scheme

Innumerable decision support systems have been conceived and implemented since time immemorial in all kinds of human affairs. Those used in environ-

[3] Of course, the distribution needs not to be gaussian

mental sciences, as for example in meteorology, have for a long time been based on observation, experience and memory. Scientific analysis, quantified observation and numerical simulation shape modern meteorological decision schemes. However, even if human forecasters refer to those comprehensive technological systems when taking decisions, they cannot help letting personal bias playing a more or less concealed role in their cogitations.

The choice of a decision scheme is already a decision *per se*, indeed a "*meta-decision*". In this perspective, seeking rationality and aimed at inhibiting any potential psychological slant, the system is implemented as a simple automatic algorithm operating in accordance to the rules given in Table 2:

Threshold	Units	$\tilde{Q} < Q$	$\tilde{Q} \geq Q$
	Event	did not occur	did occur
Probability	Alarm		
$\int_Q^B P(q)dq \geq P$	issued	False alarm	Hit
$\int_Q^B P(q)dq < P$	not issued	Correct rejection	Miss

Table 2.

Reflecting on the interplay between the probability and the meteorological thresholds, the reader will have noticed that, the latter being determined by the addressee, the former is likely to become the governing decisional parameter in the issuer's hands. Once again, economic considerations will clarify the issue. To this purpose, the issuer's profile has to be sketched first.

6.3 Drawing the issuer's profile

The definitions of the hit rate, false alarm rate and ratio introduced in the Appendix, Section 12.1. are applied again. As explained in Section 4 and Appendix 12.1, the false alarm ratio is preferred to the false alarm rate. Being the ratio of the number of mistakenly issued warnings to the total number of issued warnings, the false alarm ratio describes better the quality of the service provided by the issuer. Furthermore, the number of non-events becoming large when high meteorological thresholds are considered, the false alarm rate dwindles accordingly. On the contrary, the false alarm ratio, remaining stable, happens to be a better estimator of the issuer's performance when extreme events selected with high meteorological thresholds are considered.

The results of simulations of up to 10.000 cases are presented in Figure 14. Meteorological thresholds are $Q = 10$ and 14 units, on the top row and $Q = 18$ and 22 units, in the bottom row. Yellow - orange curves describe relative operating characteristics computed with the false alarm ratio. Green - blue curves describe relative operating characteristics computed with the false alarm rate. All curves are parametrised in probability thresholds, running from $P = 10\%$ to $P = 90\%$ in steps of 10%. These are the probabilities sketched in Figure 13 and taken into account in the decision scheme provided in Table 2, Section 6.2.

Unmistakably do green curves telescope onto the left edge of the diagrams when higher meteorological thresholds are considered. On the contrary, orange curves are stable. Remaining settled at centre stage when meteorological thresholds are high, they demonstrate the better reliability of the false alarm ratio when extreme events are at stake. In each diagram, the yellow broken line with red vertexes represents the rough results directly emanating from the simulation. The orange continuous curve is obtained after polynomial smoothing of the broken line.

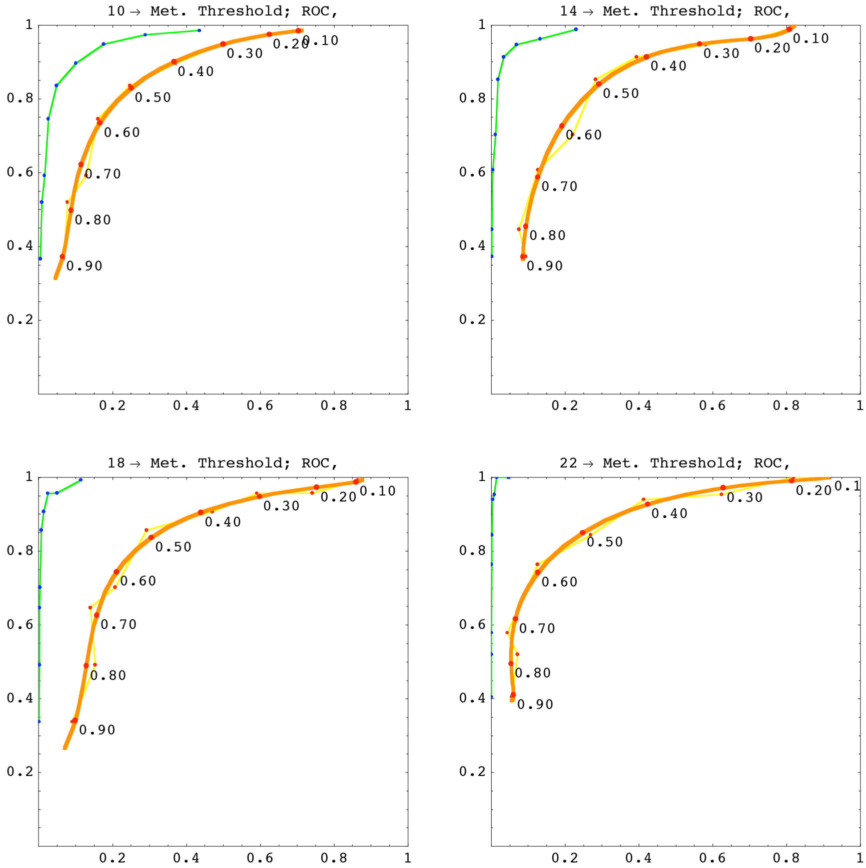

Figure 14: *Relative operation characteristics derived from simulations of the EPS. Abscissa: false alarm ratio (rate); ordinate: hit rate. Orange / yellow curves describe the Far-ROC, green curves the F-ROC. The parametrization running on each curve expresses the corresponding probability threshold between $P = 10\%$ and $P = 90\%$. Top row: meteorological thresholds 10 and 14 units. Bottom row: meteorological thresholds 18 and 22 units.*

6.4 Drawing the issuer's profile, alternative formulation

The alternative formulation presented here is based on the the distribution of the probabilities delivered by the warning system operating at a given meteorological threshold Q. Such curves are usually represented as inset boxes in reliability diagrams of forecasting systems, as in Reference [4], page 16. Represented hereafter in Figure 15, the curves $u_{(p)}$ and $v_{(p)}$ depict the distributions of probabilities delivered by a warning system operating at a given meteorological threshold Q when events did actually occur: $u_{(p)}$ distribution, accordingly when they did not: $v_{(p)}$ distribution[4].

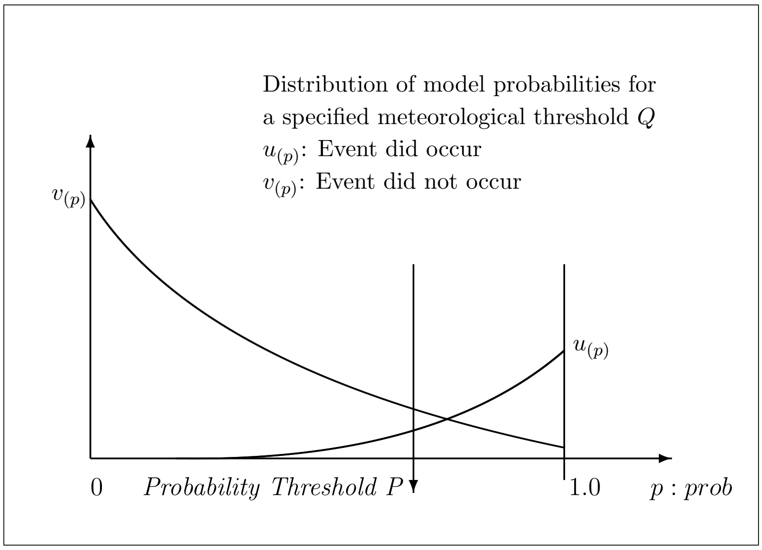

Figure 15: *Distribution of warning probabilities at a given meteorological threshold Q. Abscissa: probability at which warnings are issued, with possible probability threshold P depicted by the downward pointing vertical arrow. Ordinate: frequency of occurrences of probabilities, expressed in case of occurring, respectively not occurring events.*

The area located below curve $v_{(p)}$ in the domain $p \in [0, P[$ corresponds to the entry "a" in the table A.1 given in Section 12.1. The area located below curve $u_{(p)}$ in the domain $p \in [0, P[$ corresponds to the entry "b" in the same table. The area located below curve $v_{(p)}$ in the domain $p \in [P, 1]$ corresponds to the entry "c" in the table. The area located below curve $u_{(p)}$ in the domain $p \in [P, 1]$ corresponds to the entry "d" in the table.

A simulation based on this alternative formulation is presented in Figure

[4]These probability functions correspond to the concept of signal and noise, as described in the signal detection literature. [12]

16, where the $u_{(p)}$ and $v_{(p)}$ distributions are approximated as polynomials of degree 4.

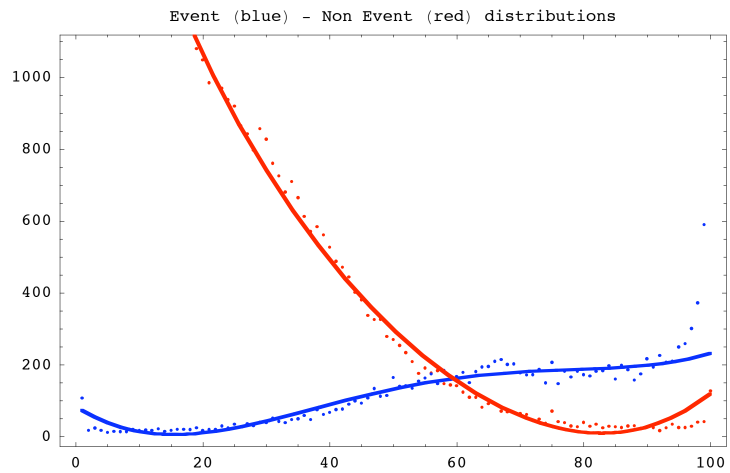

Figure 16: *Simulated distribution of warning probabilities at meteorological threshold $Q = 10$ units, corresponding to Figure 15. Abscissa: probability at which warnings are issued, expressed in percents. Ordinate: distribution of probabilities, expressing the frequency of occurrences of forecasts for occurring (blue dots), respectively not occurring events (red dots). Corresponding polynomial approximations (blue and red curves).*

Expressed in terms of the polynomial approximations $u_{(p)}$ and $v_{(p)}$, the hit rate at probability threshold P is given by the quotient of the area located below the $u_{(p)}$ curve integrated between P and 1, and the total area under that curve:

$$H_{(P)} = \frac{\int_P^1 u_{(\pi)} d\pi}{\int_0^1 u_{(\pi)} d\pi} \tag{10}$$

The false alarm ratio at probability threshold P is the quotient of the area located below the $v_{(p)}$ curve integrated between P and 1, and the total area under both $u_{(p)}$ and $v_{(p)}$ curves, integrated between P and 1:

$$Far_{(P)} = \frac{\int_P^1 v_{(\pi)} d\pi}{\int_P^1 (u_{(\pi)} + v_{(\pi)}) d\pi} \tag{11}$$

These definitions enable the computation of the relative operating characteristic presented in Figure 17. Furthermore, they will be used in Section 7.3 to derive - directly from the issuer's relative operating characteristic - the

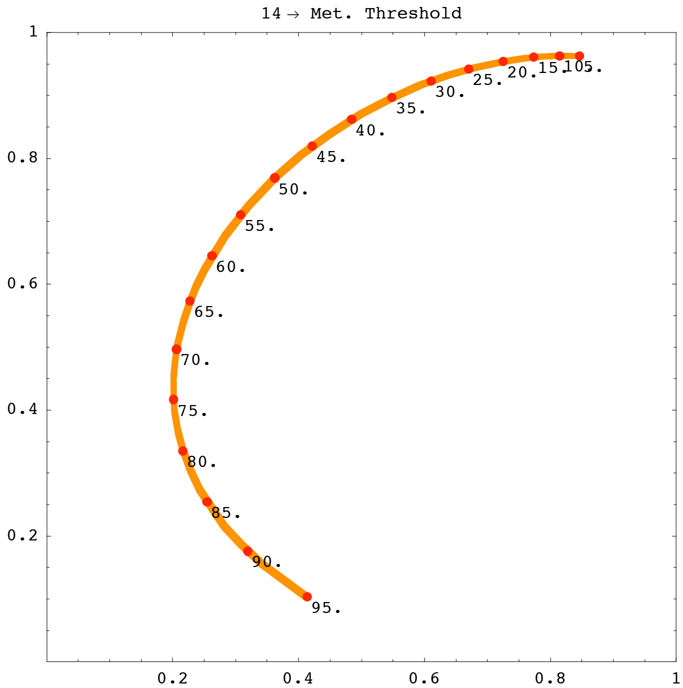

Figure 17: *Relative operation characteristic at meteorological threshold* $Q = 14$ *units based on the alternative formulation. Abscissa: false alarm ratio, ordinate: hit rate. Parametrization expressed in probability threshold* P *given in %.*

addressee's exposure at a given meteorological threshold Q and probability threshold P.

6.5 Concluding remarks

Considering the analogy with the addressee's perspective discussed in Section 5.1.2, one notices that, for one given meteorological threshold Q, low probability thresholds correspond to high hit rates and false alarm ratios, respectively high probability thresholds to low hit rates and false alarm ratios.

According to the characterisation of risk awareness presented earlier, the addressee, behaving rationally, *de facto* adopts risk adverse strategies when warnings are issued at low probability thresholds, comparatively risk friendly strategies when they are issued at higher probability thresholds. Correspondingly, issuers exhibit risk friendly behaviours when they deliver their warnings at low probability thresholds, thus tolerating high false alarm ratios.

On one side do risk adverse addressees or customers require to be alarmed early, at low probability thresholds. On the other side, risk friendly issuers tend to deliver their warnings at low probability thresholds either. This apparent contradiction will be given more attention in Section 8, Intermezzo.

7 Synthesis: Warning Decision

Having established both actors' profiles, the next step will consist in confronting them. However, the addressee's profile being expressed in hit rate and false alarm rate, the issuer's one in hit rate and false alarm ratio, a reformulation of the former is required in order to implement it into the latter's related frame. This operation is described first. The computation of the optimal probability threshold at which warnings should be issued is addressed in a following Section. Concluding remarks close the Section.

7.1 Implementing the rational addressee's profile

The elements presented in Section 5.3 remain valid and are repeated here: L represents the loss induced by a disaster for which neither a warning was issued nor mitigating measures were taken. C represents the costs induced by mitigating measures. They are due in case of occurrence of a correctly warned event, as well as in case of a mistakenly warned non-event. λ represents residual disaster costs remaining in the case of a correctly warned event.

	event did not occur	event did occur
warning issued	C	$C + \lambda$
no warning issued	0	L

Table 3.

The average costs the addressee is faced to during a period long enough to be of climatological relevance, expression (7), are:

$$M = \frac{1}{a+b+c+d}\,[bL + Cc + (C+\lambda)d] \qquad (12)$$

with a, b, c, d, H and Far as defined in Appendix, Section 12.1 and computed following the decision scheme presented in the Figure 13, Section 6.1, and Table 2, Section 6.2.

The derivation of the economic function based on hit rate and false alarm ratio is presented in the Appendix, Section 12.5.2. This function, hereafter referred to as M_A, reads:

$$M_{A(H,Far)} = L\,\Omega\,[(1-H) + H\,\Lambda + \Gamma\,\frac{H}{1-Far}] \qquad (13)$$

$$= L\,\Omega \cdot \begin{bmatrix}1 & \Lambda & \Gamma\end{bmatrix} \cdot \begin{bmatrix}1-H \\ H \\ H\,(1-Far)^{-1}\end{bmatrix} \qquad (14)$$

It can be written either as a standard algebraic expression (13), or as a scalar product (14). Disentangling the roles played by the different protagonists, the second set-up, equation (14), will be preferred for reasons explained below in this Section.

The climate burden $L\,\Omega$, defined in Section 5.1.2, equation (3), represents the average costs the addressee would face if he were to assume his climatic fate without undertaking any mitigating or protective action. As an example, were he expecting adverse weather events costing $L = 1000$ monetary units with probability of occurrence $\Omega = 0.1$ per year, then, over a long period of time, he would have to pay a yearly climate burden of 100 monetary units.

However, proactive players being considered in this study, it will be assumed that the addressee decides to rely on warnings provided by an issuer, therefore requiring his actual burden to lie well below the "fateful" climate burden. Explicitly, he will seek for the minimum between those two quantities, $L\,\Omega$ and $M_{A(H,Far)}$, the first one being governed by the climate, the second one by the performance of the warning system. Accordingly, the economic function is modified into:

$$\mathcal{M}_{A(H,Far)} = L\,\Omega \cdot Min\{1\ ,\ \begin{bmatrix}1 & \Lambda & \Gamma\end{bmatrix} \centerdot \begin{bmatrix} 1-H \\ H \\ H\,(1-Far)^{-1} \end{bmatrix}\}.$$

Moreover, considering that an insurance company shrewdly hedging its risks could offer to cover the addressee's financial exposure at a premium $\mathcal{P}$ set well below the climate burden $L\,\Omega$, the issuer would enter into competition with that company. For the addressee, the comparison would no longer occur between the climate burden and the issuer's service, but instead between the contract presented by the insurance company and the service offered by the issuer. In such a situation[5], the coefficient 1 in the $Min\{1, \cdots\}$ expression would have to be replaced by the "insurance coefficient" $\mathcal{S} = \frac{\mathcal{P}}{L\,\Omega} \leq 1$.

The rational addressee's profile is finally implemented in accordance to equation (9) and the economic profile at meteorological threshold Q, expressed in monetary units, reads:

$$\mathcal{M}_{Q(H,Far)\mathcal{S}} = L\,\Omega \cdot Min_{Q(H,Far)\mathcal{S}} \qquad (15)$$

[5]The competition could occur not only with an insurance company, but also with another warnings issuer, who could propose any other decision scheme to the addressee.

with:

$$Min_{Q,(H,Far)\mathcal{S}} = Min\{\mathcal{S}\ ,\ \begin{bmatrix}1 & \Lambda & (1-\Lambda)E_{(Q)}\end{bmatrix} \cdot \begin{bmatrix}1-H \\ H \\ H\,(1-Far)^{-1}\end{bmatrix}\}.$$

The promised disentanglement can now be explained: four building blocks are taken into consideration.

They are: 1) the climate burden $L\,\Omega$, 2) the insurance company, whose possible action is parametrized by $\mathcal{S}$, 3) the addressee's profile, described by the row vector $\begin{bmatrix}1 & \Lambda & (1-\Lambda)E_{(Q)}\end{bmatrix}$, and 4) the issuer's performance profile, described by the column vector[6] $\begin{bmatrix}1-H & H & H(1-Far)^{-1}\end{bmatrix}^T$. It is worth noticing that the cost-loss ratio does no longer explicitly appear. It has been made implicit by the requirement of rationality. The possible intervention of an insurance company being ignored in the following, the coefficient is accordingly set to: $\mathcal{S} = 1$ for all considerations and in all graphs presented hereafter.

The addressee is confronted with his climate burden when the warning system is disconnected (hit rate and false alarm ratio both set to zero): $\mathcal{M}_{Q(H=0,Far=0)\mathcal{S}=1} = L\,\Omega$. He has to pay the average mitigating and residual costs when the warning system, working perfectly, detects all events (hit rate equals to one) and issues no false alarms (false alarm ratio equals to zero): $\mathcal{M}_{Q(H=1,Far=0)\mathcal{S}=1} = (C+\lambda)\,\Omega$.

The graph of function (15) is presented in Figure 18 for an addressee operating at meteorological threshold $Q = 20$ units. The plateau visible on the rear left side of the surface corresponds to the area where, the false alarm ratio being high, the reliability of the warning system is poor and the costs induced by numerous false alarms are higher than the climate burden $L\,\Omega$. This function will be instrumental in the determination of the appropriate hit rate and false alarm ratio at which the warning system should be operated. This will be done in the next Section with the computation of an optimal probability threshold P^*.

[6]"T" means transposition.

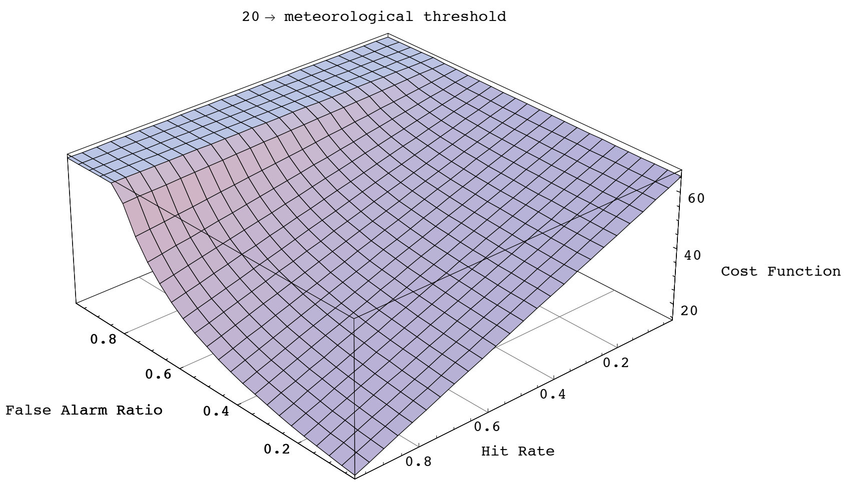

Figure 18: $\mathcal{M}_{Q(H,Far)}$, *representing the economic function. The hit rate runs along the right edge of the cube, the false alarm ratio along the left edge. The meteorological threshold is arbitrarily fixed at* $Q = 20$ *units. The plateau visible on the rear left side corresponds to high false alarm ratios and poor issuer's reliability. Indeed, the addressee is likely to deal with an insurance company or another warning provider in this is the area.*

7.2 Optimal probability threshold P^*

Profiles of the rational addressee and of the issuer are now both expressed in terms of hit rate and false alarm ratio. They can be superposed onto the corresponding relative operating characteristic (ROC) on the H, Far-frame $\{[0,1] \times [0,1]\}$, as presented in Figure 19, This figure is the exact extension of Figure 17. The network of black lines depicts the iso-costs of the rational addressee's economic function $\mathcal{M}_{Q(H,Far)}$, expression (15) introduced in Section 7.1 and presented in Figure 18. On each panel in Figures 19 and 20, the orange ROC describes the issuer's profile, the green curve the rational addressee's profile, all of them being computed at a meteorological threshold Q^* specified by the addressee. The vertical green line represents the boundary of the plateau depicted in Figure 18. As already evoked in Section 5.5, iso-costs can be interpreted as addressee's equal sensibility or iso-tolerance loci.

Following the same methodology as in Section 5.3, it is considered that the addressee will seek out his minimum financial burden in the long term, and will therefore require to be warned at that probability threshold providing the minimum value of the economic function $\mathcal{M}_{Q^*(H,Far)}$. Noticing that the rational addressee's profile is convex and the issuer's profile concave, it appears that the optimal probability threshold is naturally defined at the point of tangency of both profiles, represented by the green dot in Figure 19 and on each panel in Figure 20. Thanks to the convexity of the iso-costs and the concavity of the ROC, it is that unique point of the ROC where the economic function reaches its minimum. The determination of the optimal probability threshold P^* is based again on the risk / reward approach used in Section 5.4. This time, however, rational addressee's and issuer's profiles are taken into consideration:

$$\frac{\partial_P H}{\partial_P Far}|_{Issuer_{(Q^*)}} = \frac{\partial H}{\partial Far}|_{Addressee_{(Q^*)}}.$$

No longer as straightforward as the case discussed in Section 5.4, the derivation is presented in the Appendix, Section 12.6, and leads to the determination of P^*.

The result is geometrically presented in Figure 19. Issuer's (orange) and rational addressee's (green) profiles are provided by the corresponding curves with the optimal probability threshold represented by the green dot at probability $P^* = 44\%$.

Following Figure 20 presents the same information for four various meteorological thresholds, with the corresponding climate and exposure profiles

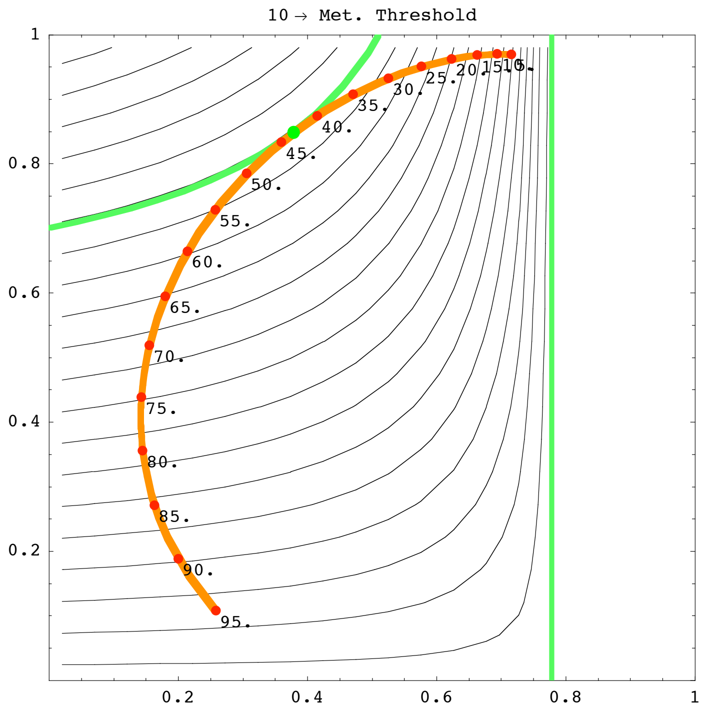

Figure 19: *Issuer's (orange) and rational addressee's (green) profiles. Abscissa: false alarm ratio; ordinate: hit rate. Meteorological threshold* $Q^* =$ *10 units with the corresponding optimal probability threshold represented by the green dot at probability* $P^* = 44\%$*. The vertical green line represents the boundary of the plateau depicted in Figure 18. Relevant for the computation of the efficiency of the warning system, it is discussed in Section 9.4.*

shown in Figure 21.

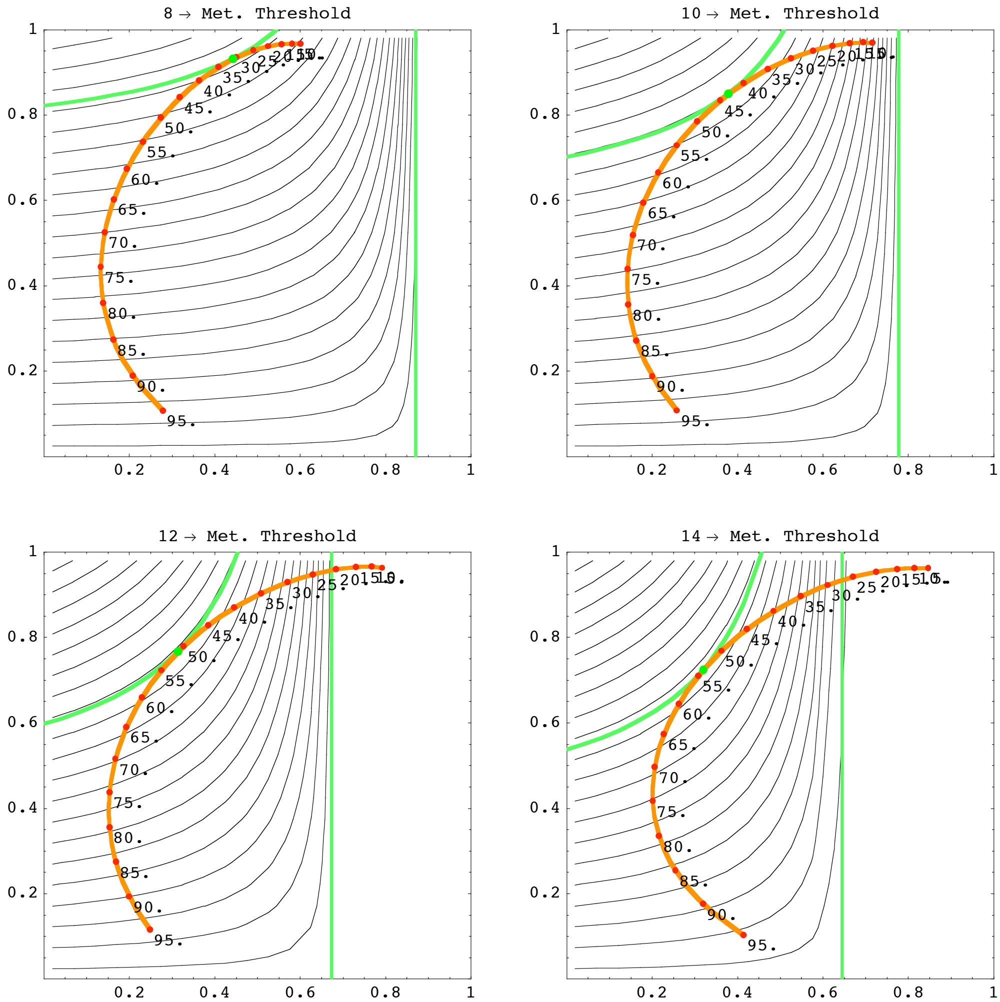

Figure 20: *Issuer's (orange) and rational addressee's (green) profiles as in Figure 19 for meteorological thresholds* $Q^* = 8$*, 10, 12 and 14 units and optimal probability thresholds.* $P^* = 31$*, 44, 51 and 54 %.*

7.3 Addressee's exposure expressed in terms of issuer's performance

A pragmatic relationship can be established between the addressee's exposure on the one side, and the performance achieved by the warning system, on the other side. Derived in the Appendix, Section 12.7, it reads:

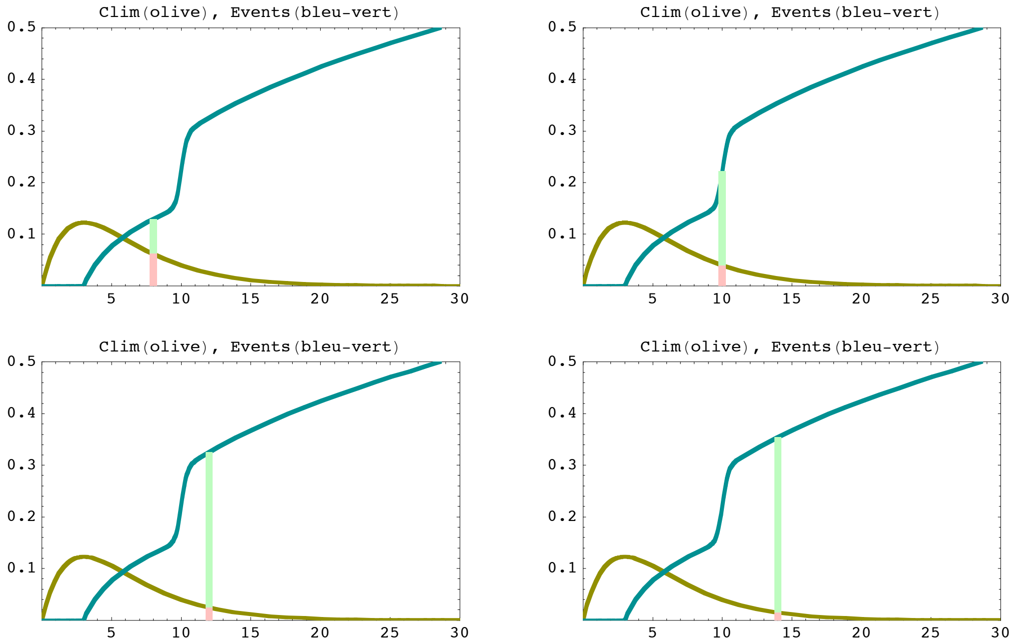

Figure 21: *Climate and exposure profiles used for the computation of the panels presented in previous Figure 20. Corresponding meteorological thresholds at* $Q^* = 8$*, 10, 12 and 14 units. The tallness of the green/pink threshold bars measures the corresponding addressee's exposure.*

$$E_{(Q^*)} = \frac{\Gamma}{1-\Lambda} = \frac{(Far-1)^2}{H\frac{\Delta Far}{\Delta H} - Far + 1}.$$

Formally equivalent to equation (9), its interpretation is made clear with an example. Let us suppose that a warning system operates at hit rate and false alarm ratio $[H = 0.8, Far = 0.4]$ with $\frac{\Delta Far}{\Delta H} = 3$. (this last figure means that the slope of the ROC at the operating point $[H = 0.8, Far = 0.4]$ is 1/3). The ratio $\frac{\Gamma}{1-\Lambda}$ can then been evaluated and gives 12%. Thus an approximation of the addressee's cost loss ratio is directly derived from the issuer's performance. It is exact if the residual loss ratio Λ is known.

Furthermore, using the definitions of the hit rate and false alarm ratio introduced in Section 6.4, equations (10) and (11), a direct connection is established between the reliability diagrams of the warning system (distributions $u_{(p)}$ and $u_{(p)}$ introduced in Section 6.4), and the rational addressee's exposure at meteorological threshold Q^*. Derived is Section 12.7.1 and graphically presented in Figure 22, the connection reads:

$$E_{(Q^*)}|_p = \frac{u_{(p)}}{u_{(p)} + v_{(p)}} \tag{16}$$

The interpolated sigmoidal approximation, (green curve in Figure 22) is thoroughly discussed in Section 8.3, Risk awareness.

Finally, disposing of reliability curves of the warning system at various meteorological thresholds, the interplay between meteorological and probability threshold, and exposure, can be broken down into three sub-conditions:

The addressee has chosen -

both meteorological and probability thresholds, then he assumes that equation 16 provides his actual exposure.

his exposure and meteorological threshold, then solving equation 16 provides the optimal probability threshold at which warnings should be issued.

his exposure and his probability threshold, then computing equation 16 at various meteorological threshold enables the evaluation of an adequate one.

This sequence of related conditions corresponds to the discussion presented in Section 5.4, in which the addressee's rationality was introduced. Corresponding equations (9) and (16) express this congruence algebraically.

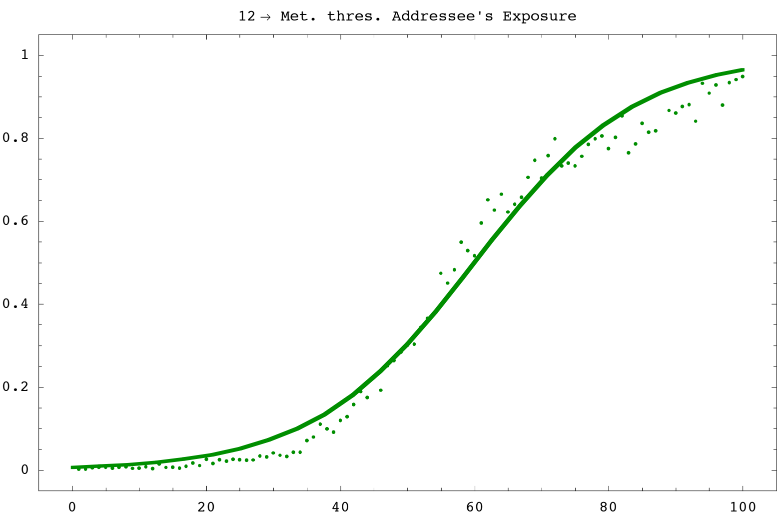

Figure 22: *Addressee's exposure* $E_{(Q^*)}|_p$ *computed with equation 16, expressed in function of the probability threshold* p *at meteorological threshold* $Q^* = 12$ *units. Abscissa: probability. Ordinate: exposure. Green dots: simulated values. Green curve: corresponding sigmoid approximation.*

7.4 Concluding remarks

As revealed through the two previous equations and demonstrated in the Appendix, Section 12.7, all economic parameters, costs, losses, residual losses, as well as the deduced parameters Γ and Λ have disappeared in the course of the computation of the optimal probability threshold P^*. Having served as scaffoldings in the elaboration of the relationship between both actors, they vanished in the derivation, eventually leading to a relationship connecting exclusively the addressee's exposure and the forecasting skill of the issuer. Consequently, the model is focused onto the issue at stake, risk management. This is an amazing outcome.

Furthermore, comparing the expression relating the addressee's risk profile to his economic profile, developed in Section 5.4:

$$\frac{\partial_Q H}{\partial_Q F}|_{Riskprofile} = \frac{\partial H}{\partial F}|_{Economicprofile}$$

with the corresponding expression relating the issuer's profile to the addressee's, developed in Section 7.2 :

$$\frac{\partial_P H}{\partial_P Far}|_{Issuer_{(Q^*)}} = \frac{\partial H}{\partial Far}|_{Addressee_{(Q^*)}}$$

and further identifying the addressee with his economic profile, one notices that the weather related risk, the addressee is confronted with, is transformed into a decision related risk, borne by the issuer.

Finally, the alternative formulation of the issuer's profile presented in Section 6.4 enables the expression of the addressee's exposure in terms of the issuer's performance. By this way, a straightforward symmetry appears between both actors, expressed firstly through the geometrical correspondence between figures 4 and 15, secondly through the reciprocity between equations (9) and (16).

8 Intermezzo: finance, media and risk awareness

Relationships with the finance and media worlds are briefly discussed in a relatively informal style in the following Sections. On the one side, developments that occurred three decades ago in financial mathematics triggered the emergence of innovative lines of products and opened highly successful markets. On the other side, the multiplication of "warning providers" on the heavily crowded media driven stage requires a reflection related to the communicative impact of the services provided by official warning issuers. Both issues were inspirational in the elaboration of the present work.

8.1 Comparison with the portfolio theory

Figure 23 sketches the "efficient frontier" (sometimes called the Markowitz frontier) considered in portfolio management. It is in startling agreement with the ROCs introduced in the previous Sections.

The expected return of an asset is plotted on the vertical co-ordinate, the standard deviation of this return, considered as the measure of the corresponding risk, on the horizontal co-ordinate. Every asset combination can be plotted in this return - risk space, and the collection of all such assets defines the region below the efficient frontier. Portfolios are constructed as combinations of individual assets. From a meteorological point of view, ensemble predictions can be assimilated to portfolios whose assets would be the members of these ensemble predictions. The analog of the addressee's profile in the financial world is called the Capital Allocation - or Market - Line.

Portfolios lying along the efficient frontier produce the highest return for a given risk or, alternatively, the lowest risk for a given return. The efficient frontier is frequently dubbed "the hedge" in financial jargon. Financial products lying on the hedge are said to be "hedged" and accordingly called "Hedge Funds". Correspondingly, inefficient portfolios and securities lie below the hedge. Thus, daring to close the analogy, optimal ensemble predictions might be compared with financial products. Best possible warning performances, instead of being provided "on the Hedge" would be settled "on the ROC". Of course, poor warning systems would accordingly be associated with inefficient portfolios and securities.

It is worth noticing that a parametrization running on efficient frontiers does not exist. They simply express the relation between return and volatility. On the contrary, both ROCs considered here are parametrized. The first one, describing the addressee's profile, is expressed in terms of meteo-

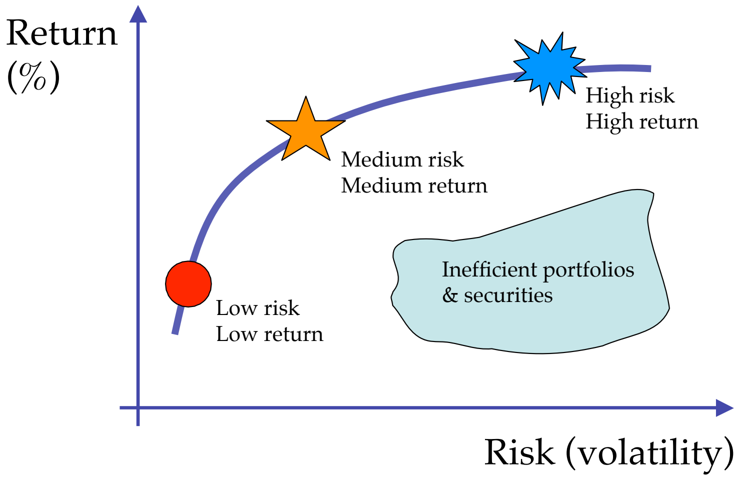

Figure 23: *Efficient Frontier as equivalent to the relative operating characteristics. Risk awareness expressed in the realm of the Hedge Fonds industry: Trade-off between risk and return. Adapted after F. S. Lhabitant, [10].*

rological threshold, the second one, related to the issuer's profile, in terms of probability threshold. This dual parametrization is instrumental in the optimal tuning of the warning system.

8.2 Competition among warning instances

A striking difference between financial and meteorological realms becomes apparent when comparing Figure 23, and Figures 6 and 10. Risky portfolios, favoured by risk friendly investors, are located on the top right end of the efficient frontier, where risk adverse addressees have been settled in our meteorological setting (aforementioned Figures). Indeed, as already suggested, the behaviour of risk friendly investors should rather be compared with that of exuberant warning issuers who would tend to "overwarn". Such an attitude naturally emerges when several warning organisms operate on a competitive basis within a media driven society.

The "Media forcing" arrow in Figure 24 depicts this trend. Research and development efforts resulting in improvements of the relative operating characteristics are sketched with the blue - red arrow. They are aimed at increasing the hit rate and reducing the false alarm ratio. Uncontrollable rivalry among risk friendly warning issuers pulls in another direction and conceals the risk of jeopardising the stability as well as the credibility of the warning system, [11].

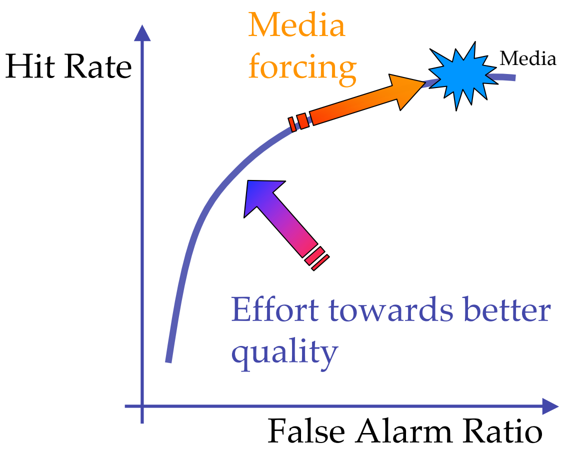

Figure 24: *Media forcing: warning issuers are inclined to operate at low probability thresholds and high false alarm ratios, thus jeopardising the credibility of a warning system.*

8.3 Risk awareness

As a matter of fact, primarily risk adverse addressees inconspicuously tend to support the trend hitherto discussed. An ultimate correction to this quite unfortunate retroaction would consist in increasing their risk congeniality! They would then simply disregard premature warnings. Modifications on disaster profiles could improve the addressee's resilience to adverse weather events as well, thus supporting their risk congeniality. On the issuer's side, legal actions could help asserting the central position of national meteorological and hydrological services on the media driven stage. Both classes of actions are in political hands. Conclusively, following Table 4 provides a summarizing overview of the dualistic risk awareness discussed so far.

If the Addressee	then the Issuer
requires early warnings, *he is risk adverse*	must warn upon weak evidence, *he is required to behave* *in a risk friendly manner*
prefers late warnings, *he is risk friendly*	can warn upon strong evidence, *he is consequently risk adverse*

Table 4.

Intuitively, two asymptotic situations should be considered. Firstly, if his cost loss ratio Γ is negligible, the addressee will decide to have the mitigating actions switched on all the time, implicitly considering the probability threshold set to zero. On the other hand, if the cost loss ratio is high, then confronted with unaffordable costs, he will require almost certainty to trigger mitigating actions.

This intuition is given a formal structure through the analysis of the sigmoid function sketched in Figure 22 and repeated in following Figure 25. Sigmoids are natural decision functions: a given action is not undertaken below a threshold, on the left branch of the sigmoid, it is on the right branch. The decision is unambiguous when the sigmoid leaps stiffly from a state to the other, as in Figure 25, it is fuzzy when the transition is smooth, as in Figure 22. Sigmoids have already been used in Section 5.1.2 (and described in the Appendix, Section 12.4.1) in order to simulate sudden financial leaps occurring in the cost multiplier function. Their natural appearance in the present context underscores the fact that warning systems are indeed decision systems. This claim is justified in the Appendix, Section 12.7.2.

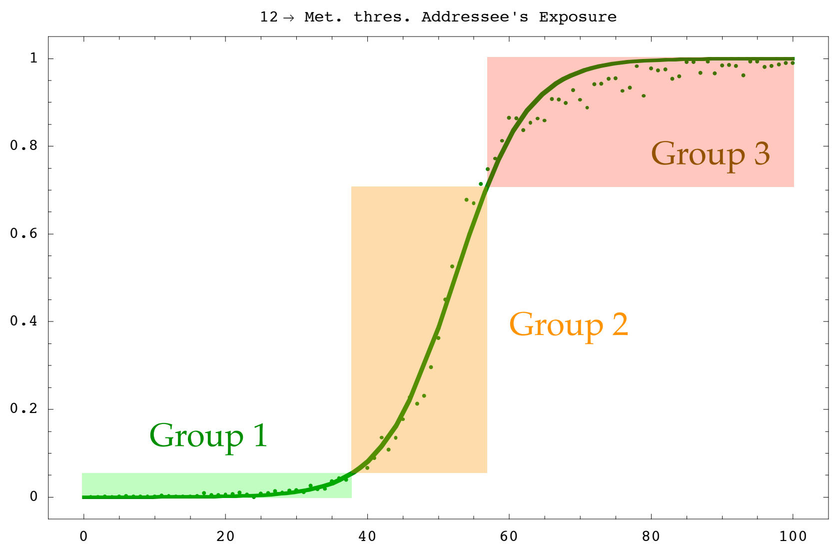

Figure 25: *Addressee's exposure computed with equation 16, expressed in function of the probability threshold p at meteorological threshold* $Q^* = 12$ *units. Three addressees families are distinguished for low, medium and high exposures, Groups 1 - 3.*

The quality of the warning system is reflected in the steepness of the sigmoidal leap. The leap is lazy in the example given in Figure 22, where the EPS simulator has been tuned in order to produce weak forecasts. On the contrary, Figure 25 has been drawn with the EPS simulator tuned in the converse direction, thus delivering reliable forecasts[7] .

The stiff sigmoid presented in Figure 25 enables a distinction between three groups of addressees, each associated with a segment of the sigmoid.

The ***first group***, located on the lower left segment, gathers addressees whose cost-loss ratios - or exposures - are almost negligible. Although being narrowly distributed on the ordinate, the spread in probabilities at which they should be warned, given on the abscissa, is high. Considering the negligible cost of mitigating actions at their disposal, as well as the indecision in probabilities, they are inclined in having their mitigating actions deployed all the time. Accordingly, they simply disregard low intensity weather warnings, whatever their probabilities. People travelling in England with an umbrella in their luggage belong to that category. Swiss drivers having snow tires mounted on their cars throughout alpine winters do too. Following the previous discussion related to risk awareness, they behave in a risk adverse manner and are likely to be satisfied with public weather bulletins.

Addresses belonging to the ***second group*** are definitely more weather sensitive. Noticing the tallness of the second rectangle, one realizes that they tend to be warned at almost similar probability thresholds, whatever their exposures. This tendency increases with the sharpness of the sigmoidal leap: where the warning system infinitely reliable, then the leap would be transformed into a step. In such conditions, the exposure profile would no longer matter. One single probability threshold would suit all addressee's requirements. Conclusively repeating the discussion related to risk awareness, one notices that addressees belonging to second group, operating at higher exposure, exhibit a lower risk aversion than their colleagues of the first group. They will require customer tailored warnings.

Addressees gathered in the ***third group*** are characterized by high mitigating costs. False alarms exert devastating consequences on their businesses. Accordingly, they require very low false alarm ratios when considering warnings, and have to establish an accurate balance between costs induced by missed events, respectively by false alarms. Indeed, they naturally tend to switch to insurances contracts or weather derivatives. Addresses whose businesses cannot be protected, as for example viticulturists against hail, are to be found in this group too. As those of the first group, addressees belonging to the third group tend to disregard warning information. Their risk aversion, however, is minimum. Instead of protecting their businesses

[7] In the former case the variance parameter u used in the $N(t, u)$ distribution (Appendix, Section 12.4.2) has simply been raised to $u = 2.5$, in the latter it has been sunk to $u = 1$ (units are irrelevant here).

and relying on warnings, they seek financial security. Both attitudes, consisting in preferring weather derivatives or insurances, can be represented in Figure 1 as transition from the upper right panel either to the left panel, or downwards to the lower right panel.

Retrospectively considering the list of cost-loss ratios presented in Section 5.3, one could infer that most of them represent addressees belonging to the first group. This interpretation is misleading. Exposures considered in Figure 25 include possible residual losses through the parameter $\Lambda > 0$, equation (9). Addresses who would belong to the first group in the classical Richardsonian set-up are therefore likely to be transfered into the second one if their residual losses are substantial.

9 Performance versus Efficiency

Experience unveils divergences in the expectations laid by our actors on the warning system they are together bound to. The primordial addressee's requirement is efficiency. The warning system has to be tuned in a way keeping the disastrous impact of adverse weather events on his business at minimum. Costs induced by false alarms are to be considered as well. From the addressee's perspective, and following the concepts introduced so far, the efficiency is to be measured in monetary units. On the contrary, the issuer is genuinely interested in reaching the maximum performance of the system he is in charge of, thus preferably considering relative operating characteristics, hit rate, false alarm ratio, or any other valuation scheme of that kind.

The dualistic approach favoured in this study enables the elaboration of quantitative measures connecting both actors' expectations. They are addressed in the following Sections.

9.1 Assessing the performance of a warning system

In this essay, the performance of the warning system is provided by the two parameters hit rate and false alarm ratio. The performance is computed at given meteorological thresholds Q^* for probabilities and $P \in [0.05 \ldots 0.95]$. ROC-curves presented from Figure 17 onward have been computed in this way.

Officially, the relative operating characteristic, noted "$\mathcal{ROC}$" is defined as:

$$\mathcal{ROC} = \int_{F=0}^{F=1} H(f)df.$$

It is a global measure of the performance of a warning system, encompassing all probability thresholds and defined as the area measured below the {H,F} ROC curve for false alarm rates f running from $F = 0$ to $F = 1$. However, remembering the discussion presented in Section 6.3 and Figure 14, this formulation of the performance is disregarded in this essay.

9.2 Assessing the efficiency of a warning system

The efficiency measure relies on the addressee's profile, given by the expression $\mathcal{M}_{Q^*(H,Far)\mathcal{S}}$, equation (15). Its derivation is presented in the Appendix, Section 12.7.

The efficiency of a warning system is expressed as the ratio between two monetary quantities: $\mathcal{F} = \dfrac{\Delta_{warn}}{\Delta_{max}}$, where:

Δ_{warn} is the monetary difference between the climatic burden $L\,\Omega$ and the ***warned burden***, defined as $\mathcal{M}_{Q^*(H,Far)\mathcal{S}}$.

Δ_{max} is the maximum possible value of this difference.

Formally, the efficiency depends on the hit rate as well as the false alarm ratio at a given meteorological threshold Q^*. Taking into account the presence of the $\mathcal{S}$ parameter, the eventual influence of an insurance company could be considered as well. It reads:

$$\mathcal{F}_{Q^*(H,Far)\mathcal{S}} = \frac{1 - Min_{Q^*(H,Far)\mathcal{S}}}{1 - Min_{Q^*(1,0)\mathcal{S}}} \tag{17}$$

One notices that the climate burden does no longer intervene in this definition. Numerically, the efficiency $\mathcal{F}_{Q^*(H,Far)\mathcal{S}}$:

equals zero: 1) when the warning system brings no positive departure in the addressee's financial burden from his maximal (climate) burden.
2) when the warning system is disconnected, with both hit rate and false alarm ratio set to zero (according to the definition of $\mathcal{M}_{Q^*(H,Far)\mathcal{S}}$, equation (15)).

equals one: when the warning system, working perfectly, detects all events (hit rate equals one) and issues no false alarms (false alarm ratio equals zero).

9.3 Interplay between performance and efficiency

Figure 26 illustrates the concepts yet introduced and reveals the interplay between performance and efficiency. In that figure and in Figure 27 the performance is measured on the horizontal bottom square, the efficiency in the vertical dimension.

The meteorological threshold Q^* is set at 10 units. The issuer's profile and his ROC are drawn on the floor of the cube with the green dot representing the optimal probability threshold P^*, occurring at 43 %. The vertical dimension of the cube represents the difference $L\,\Omega - \mathcal{M}_{Q^*(1,0)}$ normalised between 0 and 1 following equation (17). The climatic burden $L\,\Omega$ is at the ceiling and the costs induced by a perfect warning system, $\mathcal{M}_{Q^*(1,0)}$, on the floor. The curtain vertically unfolded in space depicts the quantity $\mathcal{M}_{Q^*(H,Far)} - \mathcal{M}_{Q^*(1,0)}$ normalised between 0 and 1. It is computed on the ROC for (non-optimal) probability thresholds between 0.05 and 0.95. The correspondence of its minimum with the location of the probability threshold P^* testifies to the validity of the optimisation scheme implemented.

The downward pointing blue arrow materialises the efficiency of the warning organisation $\mathcal{E}_{Q^*(H,Far)} = 70\%$ at the optimal probability threshold $P^* = 43\%$. The red arrows drawn on the ceiling sketch the corresponding hit rate = 85% and false alarm ratio = 38%.

The four cubes in Figure 27 illustrate the variation of the parameters presented in Figure 14 for meteorological thresholds Q^* set at 8, 10, 12 and 14 units. The fall of the performance and efficiency at high meteorological thresholds rises the issue of the selection of an optimal meteorological threshold. On the two lower panels, the red dots glued at the ceiling reveal the fact that, the efficiency of the issuer being zero for those very low probabilities and very high false alarm ratios, the addressee should prefer to settle on an insurance contract.

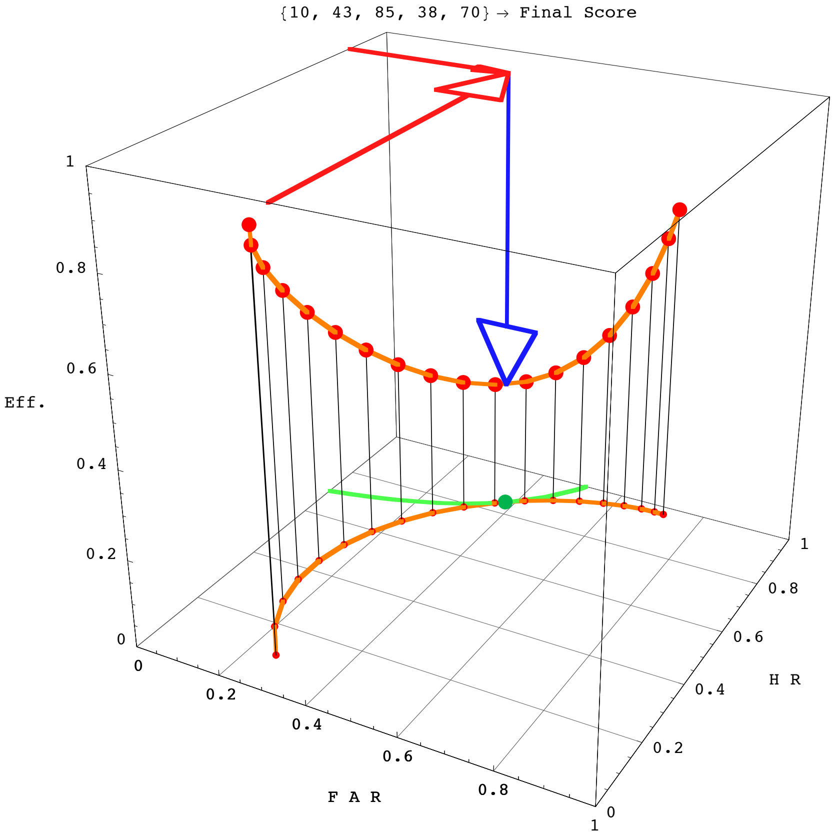

Figure 26: *Performance and efficiency indicators of the optimally tuned warning system. Abscissa: false alarm ratio; ordinate: hit rate; Vertical: normalised efficiency range (Eff.), with zero efficiency on the top, maximum efficiency on the bottom of the cube. Figures given in the brackets are: meteorological threshold* $Q^* = 10$ *units, probability threshold* $P^* = 43\%$*, hit rate* $= 85\%$*, false alarm ratio* $= 38\%$*, both pictured as red arrows. The efficiency of the warning system, pictured as the downward pointing blue arrow, is* $\mathcal{E} = 70\%$*.*

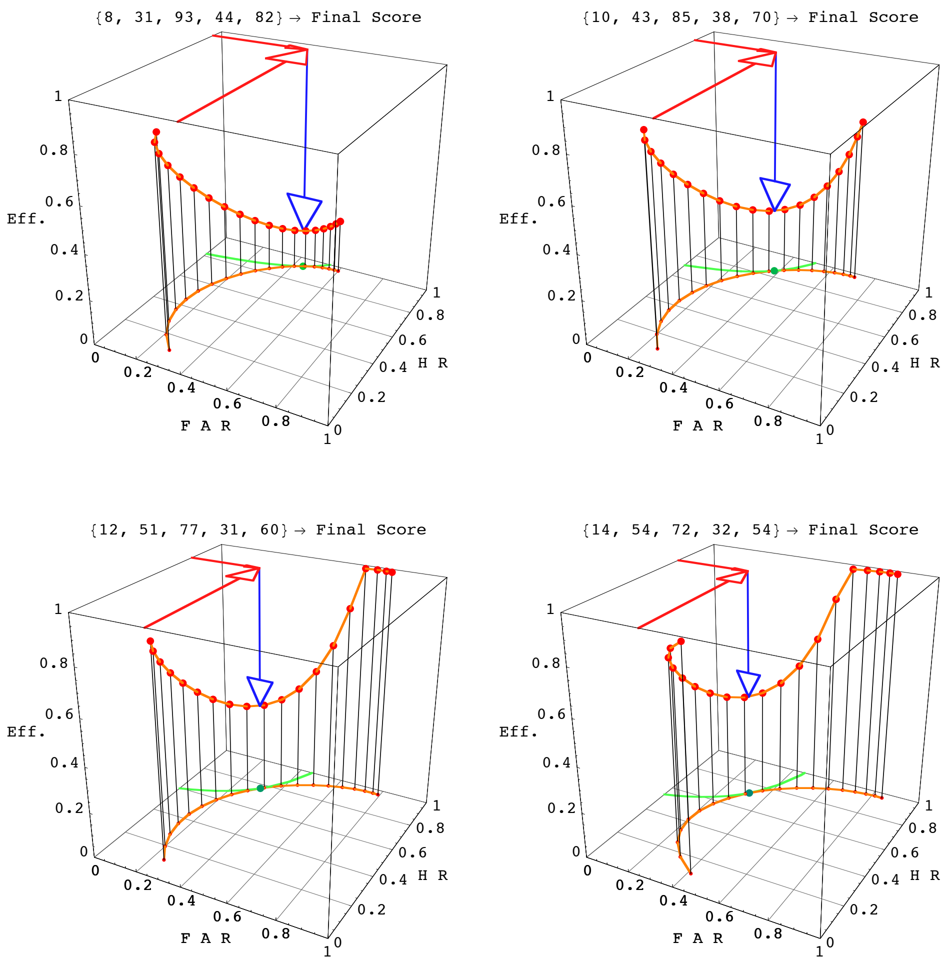

Figure 27: *Performance and efficiency indicators of the of warning system as in Figure 25, optimally tuned at thresholds* $Q^* = 8, 10, 12$ *and* 14 *units. Figures given in the brackets are: meteorological thresholds, expressed in units, probability thresholds, hit rates, false alarm ratios and efficiencies.*

9.4 Alternative presentation of efficiency and exposure

Let us now describe two graphical representations of efficiency and exposure, presented in Figures 28 and 29.

Firstly, the issuer's green curve intersects the ordinate at a value of the hit rate that is equal to the efficiency $\mathcal{F}_{Q^*(H,Far)}$ of the warning system. Following equation (17), $\mathcal{F}_{Q^*(H,Far)}$ is constant along an iso-cost curve, that could indeed be named iso-efficiency curve. It is demonstrated in the Appendix, Section 12.8, that $\mathcal{F}_{Q^*(H,Far=0)} = H$. By this way, the iso-costs (-efficiency) curves pictured in green in all figures from Figure 19 onward directly point at $Far = 0$ onto the efficiency $\mathcal{F}_{Q^*(H,Far)}$ of the warning system.

Secondly, it is also demonstrated in the Appendix, Section 12.8, that the vertical boundaries drawn in green in in all figures from Figures 19 onwards are located at abscissa $Far = 1 - E_{(Q^*)}$. A direct relationship is therefore established between the addressee's exposure and the false alarm ratio at which the efficiency of the warnings system vanishes, as depicted by the plateau drawn in Figure 18.

These elements are presented in Figure 28 for our simulated warning system, operated at meteorological threshold $Q^* = 12$ units. The vertical blue arrow depicts the efficiency $\mathcal{F}_{Q^*(H,Far)}$, the horizontal blue arrow represents the addressee's exposure, $E_{(Q^*)}$. As usually, the optimal probability threshold, located at the tangency point of the addressee's efficiency and issuer's relative operating characteristic curves, is pictured as a green dot set here at $P^* = 51\%$.

The climate burden does not appears in the computation of both efficiency and exposure. However, the warning system (included its underlying observing and numerical components), remains subjected to the climate of the area on which it is operated. This observation will be discussed further in Section 10, Interplay between climate and finance.

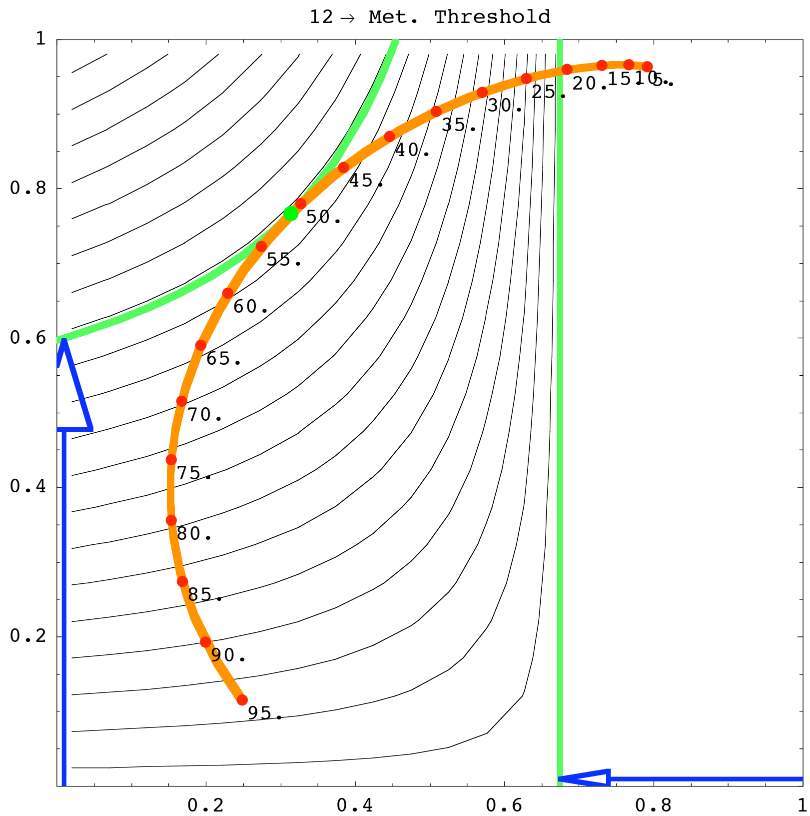

Figure 28: *Alternative graphical presentation of efficiency, provided by the vertical blue arrow, and exposure, depicted by the horizontal arrow. Meteorological and probability thresholds* $Q^* = 12$ *units and* $P^* = 51\%$ *with efficiency* $\mathcal{F} = 60\%$ *(vertical arrow) and exposure* $E_{(Q^*)} = 33\%$ *(horizontal arrow). Natural relationships between hit rate and efficiency, on the ordinate, and between false alarm ratio and exposure, on the abscissa, are persuasively pictured.*

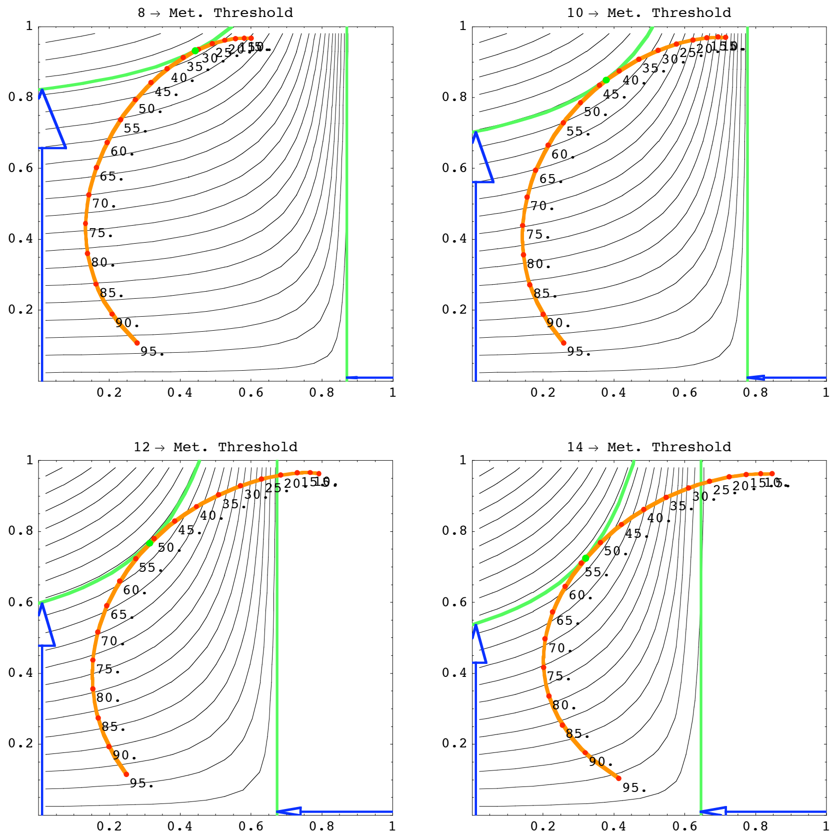

Figure 29: *Efficiency and exposure indicators (vertical, respectively horizontal arrows) of the of warning system as in Figure 27, optimally tuned at thresholds $Q^* = 8$, 10, 12 and 14 units. One notices the correspondence between the exposures given by the horizontal blue arrows and the tallness of the green/pink threshold bars plotted on the panels in Figure 21.*

9.5 Concluding remarks

The difference $\Delta_{warn} = L\Omega - \mathcal{M}_{Q^*(H,Far)}$ between the climate burden and the "warned burden", expressed in monetary units, enables the valuation of the efficiency, or the impact, of the warning system onto the addressee's economic outcome. It is to be interpreted twofold. Firstly, it can be presented to the supervising ministerial body of a national meteorological service as a proof of the economic or societal impact provided by that meteorological service. Secondly, in case of commercial business, the fee to be paid in monetary units by the addressee to the issuer can be directly computed as a proportion expressed in percents of that difference: $Fee_{\{Addressee \longmapsto Issuer\}} = x\% \; \Delta_{warn}$. This way of proceeding is in perfect accordance with the use prevailing in financial business.

For those meteorological services following the tenets of the New Public Management (NPM), impact - or efficiency - rather than performance figures might be implemented into the operative agreements settled with their supervising ministerial bodies. Indeed, the measure of the impact a meteorological service exerts onto the community it is responsible for is in perfect accordance with the philosophy NPM is aimed at.

Furthermore, fee schemes based on efficiency might be used in the elaboration of the contracts or service level agreements established between the profit centres of that meteorological service and and their customers.

10 Interplay between climate and finance

All instruments required to simulate a modification of the climate are, conceptually and computationally, available. The impact of such a modification on the efficiency of the warning system is addressed now, as well as its subsequent monetary issues.

In the following, the "reference climate", is the climate profile used until now, introduced in Figures 2 and 5 and defined as a Gamma distribution whose shape and scale parameters were arbitrarily set at $(\alpha, \beta) = (2, 3)$. It is sketched again in Figure 30. A new, "modified climate", is presented in Figure 32, defined as a Gamma distribution whose shape and scale parameters are now set at $(\alpha, \beta) = (3, 3)$. Both are sketched as usual with olive curves. Figure 32 shows that the modified climate is slightly "warmed up". By this modification, severe weather events with potentially devastating consequences are expected to occur at increased frequency. Accordingly, the addressee is faced with an amplification of the impact of such events onto his business. However, comparing Figures 30 and 32, one notices that the addressee's exposure (green curves) remains arbitrarily unchanged in this simulation.

The corresponding tuning of the warning system is presented in Figures 31 and 33, both at meteorological threshold $Q^* = 14$ units. As expected, the addressee's profile remains unchanged with the horizontal blue arrow pointing in each figure to the corresponding exposure, given here by $Far_{(Q^*=14)} = 1 - 0.35$. Oppositely, the simulated warning system happens to cope favourably with the modified climate, the corresponding ROC being slightly translated towards the upper left corner of the diagram. The efficiency rises accordingly from 57 to 64%. The probability threshold slides from 53 to 50%, thus showing a tiny increase in the issuer's risk inclination. Such a trend might indeed be expected if the issuer happens to be more frequently confronted with extreme events.

In each Figure 30 and 32, the dotted deep blue, respectively pink curves represent the dependence of the efficiency, respectively of the (scaled) warned burden (held by the addressee and expressed in monetary units) on the meteorological threshold, for the optimally tuned warning system. The former is numerically computed using expression (17), the latter expression (15).

Considering the reference climate first, Figure 30, one notices that the efficiency (deep blue curves) falls to zero for meteorological thresholds set at and beyond 20 units. As already mentioned, the efficiency is improved under modified climate, as shown in Figure 32. The comparison between the dotted purple curves, representing in both figures the warned burden, shows a dramatic financial impact of the simulated climate change onto the addressee's business. This impact is discussed hereafter in relationship with Figure 34.

Two observations are relevant in that figure. Firstly, as already noticed, the efficiency of the warning system, pictured in both panels by the dotted blue curves, increases when the climate is modified. Indeed, the relevance of an accurate warning instrument gains in importance for addresses and customers submitted to an increasingly threatening climate. Secondly, the induced costs, pictured in both panels by the dotted purple curves, rise dramatically. In this simulation, at meteorological threshold $Q^* = 14$ units, they increase by an amount of 44%. Although the efficiency of the warning system raises with the modification of the climate, it does not match the magnification of the costs supported by addressee.

In this example, where the addressee decides to bring his weather induced expenses back to those prevailing before the modification of the climate, he is bound to trigger his mitigating actions at a much lower meteorological threshold, found here at $Q^* = 10$ units, as depicted in the right panel of Figure 34. Following the discussion related to the risk awareness presented in Section 8.3, being warned at a lower threshold, he has increased his risk adversity.

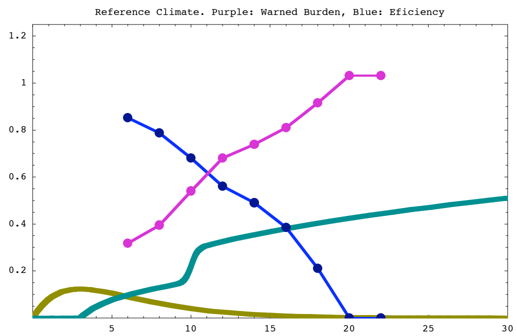

Figure 30: *Reference climate. Efficiency (deep blue) and warned burden (purple) for meteorological thresholds between 6 and 22 units.*

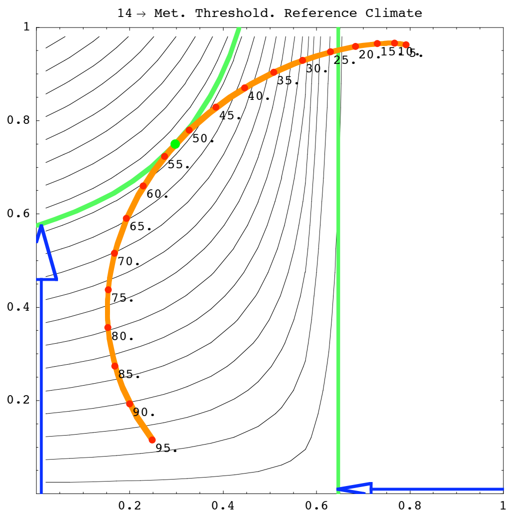

Figure 31: *Reference climate. Efficiency and exposure (arrows) at meteorological threshold $Q^* = 14$ units.*

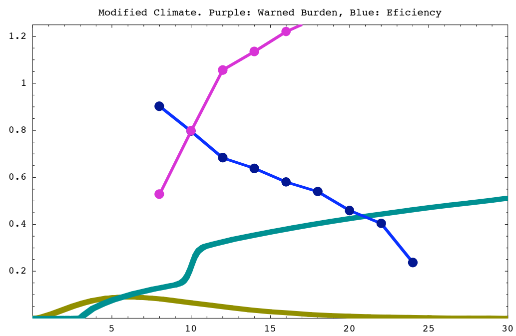

Figure 32: *Modified climate. Efficiency (deep blue) and warned burden (purple) for meteorological thresholds between 8 and 24 units.*

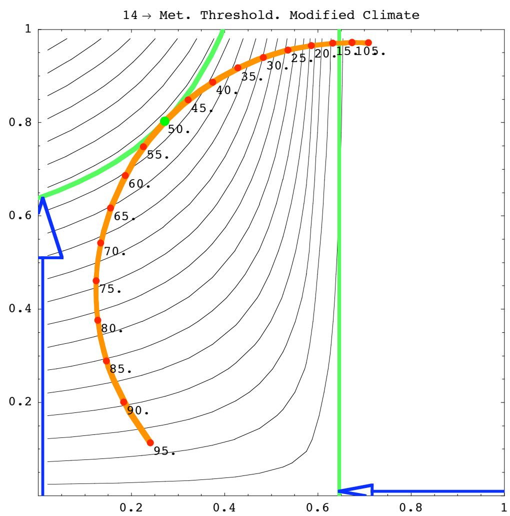

Figure 33: *Modified climate. Efficiency and exposure (arrows) at meteorological threshold $Q^* = 14$ units.*

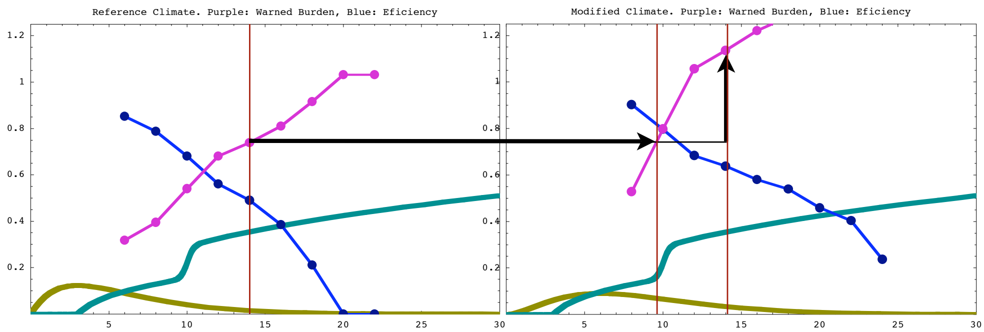

Figure 34: *Comparison between reference (left panel) and modified climate (right panel). In the left panel, where the meteorological threshold is set at* $Q^* = 14$ *units. the warned burden accounts for 0.75 (scaled) monetary units, given by the elevation of the horizontal black arrow. In the right panel, the modified climate raises the warned burden to 1.08 (scaled) monetary units, located at the tip of the vertical black arrow. The addressee reduces his warned burden to its original value (reference climate), in choosing a lower meteorological threshold located at* $Q^* = 10$ *units, at the tip of the horizontal black arrow.*

10.1 Concluding remark

Were the addressee to follow a more sensible strategy, he would manage to reduce his exposure, for example in modifying infrastructures imperilled by adverse or extreme weather, thus making them more resilient. Such an alternative perspective would allow him to keep his initial risk awareness and meteorological threshold. By this way, he would be better prepared to the confrontation with a modified climate.

11 Conclusions

The first and foremost contribution of this study is conceptual. Accordingly, possible applications should have to be refined and tailored to specific addressee's requirements and issuer's capabilities. Such a course of action is not unusual. Considering for example the mathematical game theory or the capital asset pricing model in finance, devised decades ago, one notices that both theories opened avenues that first challenged, then transformed the way people understood the problems and issues they were confronted with. Algorithms, applications, services and products were introduced consecutively.

Operating on a definitely more modest footing, the present work is aimed at clarifying the relationship between warning institutions on the one side and communities or organisations submitted to meteorological or climatic hazards on the other side. Inspired by the notion of duality that kindled the formal thinking in mathematics, physics and economics during the second half of the twentieth century, it is aimed at disentangling the addressee's and issuer's roles.

The first one, the addressee, is described by a disaster profile, measuring his exposure to adverse weather events, and an economic profile, assessing the monetary impact of those events. Both profiles are reciprocally asserted in a way enabling the computation of the meteorological threshold at and beyond which mitigating actions should be undertaken. This reciprocal assertion enables the definitions of the addressee's rationality, as well as his risk awareness.

Comparatively, the issuer's profile is straightforward. It describes the performance of a warning system operating under the meteorological constraints prevailing for the addressee. Forecasts are expressed probabilistically in terms of occurrence of weather events whose intensity lies at or beyond the meteorological threshold.

The confrontation of both addressee's and issuer's profiles then enables the computation of that optimal probability threshold at and beyond which mitigating actions should be triggered, according to the meteorological threshold.

A striking characteristic of the proposed model is its ability to directly relate the addressee's disaster profile to the issuer's performance profile. Financial or monetary considerations, having served as scaffolding in the elaboration of the model, simply vanish at the the final stage of its set-up.

However, the methodology enables the definition of the addressee's climate burden, the average costs he would be faced with if he were to operate his business without warning system. Correspondingly, the warned burden - average costs due when operating with a warning system- is evaluated. Thus the departure from the climate to the warned burden provides a measure of the efficiency of the warning system. By this way, performance objectives defined on the issuer's side are translated into efficiency impact onto the addressee's business.

Simulation of modified climate profiles, on average as well as in spread, are introduced. Enabling the computation of optimal meteorological warning thresholds under modified climatic conditions, they provide either a measure of the financial impact of such climatic modifications on the addressee's business, or a valuation of a meteorological threshod by which the financial impact of a modified climate would be compensated. Both are considered under optimal tuning conditions of the warning system.

12 Appendixes

12.1 Basic definitions of hit rate, false alarm rate and ratio

The classical contingency table describing the four possible occurrences where: a) neither did an event occur nor was an alarm raised, b) an event occurred without alarm, c) an alarm was raised although no event occurred and d) an event occurred for which an alarm was correctly raised, is reproduced in Table A.1. The four figures a, b, c, d represent the number of cases accumulated in each category during an assessment period.

	event did not occur	event did occur	sum
alarm raised	c	d	c+d
no alarm raised	a	b	a+b
total	a+c	b+d	a+b+c+d

Table A.1

The following definitions are well known: hit rate (also called Probability of Detection): $H = \frac{d}{b+d}$. false alarm rate (also called Probability of False Detection): $F = \frac{c}{a+c}$. false alarm ratio (no other name): $Far = \frac{c}{c+d}$. Frequency of occurrence of the event: $\Omega = \frac{b+d}{a+b+c+d}$.

As explained in the main text, besides the hit rate, which is of paramount importance for the addressee as well as for the issuer, the difference between the false alarm rate and the false alarm ratio is subtle and should not be underestimated. It is explained hereafter with the following example:

	event did not occur	event did occur	sum
alarm raised	100	40	140
no alarm raised	850	10	860
total	950	50	1000

Table A.2

The false alarm rate, computed on "no events", in this case not bad with $\frac{100}{950}$, measures the frequency at which the business of the addressee, instead of running smoothly, is impeded by unnecessary protective actions taken under fair weather conditions.

The false alarm ratio provides a measure of the quality of the service provided by the issuer. It is expressed in terms of the number of mistakenly issued warnings divided by the total number of issued warnings. With $\frac{100}{140}$ in this example, it is quite a poor.

The false alarm rate is an addressee's issue, the false alarm ratio an issuer's concern. The former is a measure of the efficiency of a service, the latter a measure of the performance of that service.

12.2 Derivation of the probabilistic hit rate and false alarm rate

Taking into account the concepts introduced in Section 5.1.1, the contingency table is now formulated using the probabilistic distributions representing the climatology: $C(q)$, and the frequency of occurrence of disastrous events: $E(q)$, Table A.3:

Domain	event did not occur	event did occur	sum
$[Q, B]$	$\int_Q^B (1 - E_{(q)})C_{(q)}dq$	$\int_Q^B E_{(q)}C_{(q)}dq$	$\int_Q^B C_{(q)}dq$
$[0, Q]$	$\int_0^Q (1 - E_{(q)})C_{(q)}dq$	$\int_0^Q E_{(q)}C_{(q)}dq$	$\int_0^Q C_{(q)}dq$
$[0, B]$	$1 - \Omega$	Ω	1

Table A.3

Being quite awkward, the interpretation of this table, formally corresponding to the tables introduced in Section 12.1, is accomplished with utmost care below. As a matter of definition, the term "case" used below refers to the lapse of time Δ defined in Section 5.1.1, lasting a hour, a day, even a week, during which a disastrous event might occur, or not. This disastrous event is exclusively triggered by the weather.

For a given case whose weather condition is Q, the probability of occurrence of a disastrous event is given by $E_{(Q)}$. The probability for the weather state to be equal to Q being determined by the climatic distribution $C_{(Q)}$, the probability of occurrence of a disastrous event triggered by the weather at value Q is therefore $E_{(Q)}C_{(Q)}$. Then, the integral $\int_{Q_1}^{Q_2} E_{(q)}C_{(q)}dq$ measures the probability to experience a disastrous event when the weather happens to occur between two meteorological bounds $Q_1 < Q_2$.

The upper row of the Table A.3, labelled $[Q, B]$, represents the cases at and beyond the threshold Q, for which warnings are issued and mitigating actions undertaken. The middle row of the table, labelled $[0, Q]$, represents those cases where the value of the meteorological parameter lies beneath the threshold and no warnings are issued. The lower row of the table, labelled $[0, B]$, spans all the cases where warnings are, respectively are not issued.

Let us now chase in the table and consider first the entry $\{[Q, B],$

event did occur} of value $\int_Q^B E_{(q)}C_{(q)}dq$. This integral between Q and B measures the probability of occurrence of disastrous events within this domain, where warning are issued. Below, the integral $\int_0^Q E_{(q)}C_{(q)}dq$ measures the probability of occurrence of disastrous events within the complementary domain $[0.Q]$ in which no warnings are issued. The sum of both, $\Omega = \int_0^B E_{(q)}C_{(q)}dq < 1$, measures the overall probability of occurrence of weather induced disasters. It represents the climatic component in the climate burden $L\,\Omega$ discussed in the main text.

Considering now the entry $\{[Q, B]$, event did nor occur$\}$ of value $\int_Q^B(1 - E_{(q)})C_{(q)}dq$, one notices that it can be written $\int_Q^B C_{(q)}dq - \int_Q^B E_{(q)}C_{(q)}dq$. The first integral in the entry $\{[Q, B]$, sum$\}$ represents the probability of occurrence of weather within the corresponding domain, disregarding any disastrous consequence. The difference between both integrals therefore measures the frequency of the cases where warnings are issued although no disasters occur. Accordingly, the entry $\{[0, Q]$, event did not occur$\}$ with $\int_0^Q(1 - E_{(q)})C_{(q)}dq$ measures the frequency of the cases where neither was warnings issued nor did an event occur.

The bottom row $[0, B]$ gives for both cases where disastrous events occur, or not occur, the corresponding sums expressed in terms of Ω. The bottom right corner is the sum on the last column as well as on the last row. It expresses the standard propriety of a probabilistic distribution: $\int_0^B C_{(q)}dq = 1$.

Conclusively, Tables A.1 and A.3 having the same structure, according to their definitions given in Section 12.1, the two following ratios are directly read on Table A.3. They are:

Hit rate: expresses the ratio between the probability of disasters occurring in the domain $[Q, B]$, for which mitigating actions are undertaken and the overall probability of occurrence of weather induced disasters. Following the definitions given in Section 12.1 and taking into account the correspondence between Tables A.1 and A.3, $\int_Q^B E_{(q)}C_{(q)}dq$ is substituted for d and Ω is substituted for $b + d$, thus giving:

$$H_{(Q)} = \frac{1}{\Omega}\int_Q^B E_{(q)}C_{(q)}dq.$$

False alarm rate: expresses the ratio between the probability of non occurrence of disasters in the domain $[Q, B]$ for which mitigating actions are inadequately undertaken and the overall probability of occurrence of weather conditions not triggering disasters. Following the definitions given in Section 12.1 and taking into account the correspondence between Tables A.1 and A.3, $\int_Q^B (1-E_{(q)})C_{(q)}dq$ is substituted for c and $1-\Omega$ is substituted for $a+c$, thus giving:

$$F_{(Q)} = \frac{1}{1-\Omega}\int_Q^B (1-E_{(q)})C_{(q)}dq.$$

The variable of these functions, the meteorological threshold Q, is an integration bound on the right hand side of each expression. This propriety will be given particular attention in following Section 12.3.

12.3 Derivation of the relationship between Γ, Λ and Q

The derivation is carried through as an optimisation problem. The economic profile represents the utility function to be minimised under the constraint provided by the risk profile, expressed as the relative operating characteristic. Both sides of the following expression therefore have to be evaluated:

$$\frac{\partial_Q H}{\partial_Q F}|_{Riskprofile} = \frac{\partial H}{\partial F}|_{Economicprofile}$$

Risk profile the propriety of the derivative of an integral with respect to its integration bound Q : $\partial_Q \int_Q^B f(q)dq = -f_{(Q)}$, is used here, yielding:

$$\begin{aligned}\frac{\partial_Q H}{\partial_Q F}|_{Risk} &= \frac{\partial_Q[\frac{1}{\Omega}\int_Q^B E_{(q)}C_{(q)}dq\ ]}{\partial_Q[\frac{1}{1-\Omega}\int_Q^B (1-E_{(q)})C_{(q)}dq\ ]}\\ &= \frac{\frac{1}{\Omega}E_{(Q)}C_{(Q)}}{\frac{1}{1-\Omega}(1-E_{(Q)})C_{(Q)}}\\ &= \frac{E_{(Q)}}{1-E_{(Q)}}\cdot\frac{1-\Omega}{\Omega}.\end{aligned}$$

.

Of course, $E_{(Q)} < 1\ \forall Q \in [0,B]$ has to be satisfied and $\Omega \in [0,1)$ assumed (Appendix, Section 12.2). In such circumstances, $\frac{\partial_Q H}{\partial_Q F}|_{Risk} > 0$. Furthermore, $\partial_Q(\frac{\partial_Q H}{\partial_Q F}|_{Risk}) = \frac{E'_{(Q)}}{(1-E_{(Q)})^2}\cdot\frac{1-\Omega}{\Omega} > 0$ for $E'_{(Q)} > 0$. These two requirements, $E_{(Q)} \in [0,1)$ and $E'_{(Q)} > 0$, warrant the concavity of the ROC and therefore, together with the linearity of the iso-costs, the unicity of the solution of the optimisation problem.

Economic profile Taking into account the continuity of the Richardson function with respect to its variables H and F, the theorem of the implicit function is applied in the sense that the iso-costs of the Richardson function (8) are its implicit functions. On any iso-cost of monetary value $M_\$$ the relation $M_R(H,F) = M_\$$ holds for the corresponding H and F values. Thus, if H is the implicit function expressed in terms of F, the total derivative is:

$$D_F M_R(H_{(F)}, F) = \frac{\partial M_R}{\partial H}\frac{\partial H}{\partial F} + \frac{\partial M_R}{\partial F} = 0.$$

Therefore, after few algebraic manipulations, the result reads (with "*Eco*" standing for "*Economicprofile*"):

$$\begin{aligned} \frac{\partial H}{\partial F}|_{Eco} &= -\frac{\partial M_R}{\partial F}(\frac{\partial M_R}{\partial H})^{-1} \\ &= \Gamma L \frac{\Omega - 1}{\Omega(\lambda + L(\Gamma - 1))} \\ &= \frac{1-\Omega}{\Omega} \cdot \frac{\Gamma}{1 - \Lambda - \Gamma}. \end{aligned}$$

Synthesis Requiring the identity of the slopes at the point on the ROC with curvilinear abscissa Q and accordingly equating the two quantities:

$$\frac{\partial_Q H}{\partial_Q F}|_{Risk} = \frac{\partial H}{\partial F}|_{Eco}$$

leads to:

$$\frac{E_{(Q)}}{1 - E_{(Q)}} \cdot \frac{1-\Omega}{\Omega} = \frac{1-\Omega}{\Omega} \cdot \frac{\Gamma}{1-\Lambda-\Gamma}$$

and after few further manipulations to:

$$E_{(Q)} = \frac{\Gamma}{1-\Lambda}.$$

Indeed are these concepts embedded in the broader realm of the Lagrange Multipliers and issues related to constraint optimisation. Fortunately, in the present setting the derivation of the relationship between Γ, Λ and Q is simple and does not require the mobilisation of such a sophisticated weaponry.

12.4 Simulated items

12.4.1 Addressee's exposure

In the example discussed in this work, the addressee's disaster profile is simply defined as:

$$\mathcal{E}_{(q)} = \begin{cases} 0 & q < B_1 \\ A\ ln(q - B_1 + 1) & q \geq B_1 \end{cases}$$

where A is an arbitrary multiplicative factor, B_1 the meteorological threshold at and beyond which weather events do actually matter for the addressee, satisfying $0 < B_1 \leq B$. B is the upper meteorological bound defined in Section 5.1.1.

The loss multiplier is constructed as a sequence of two steps provided by sigmoid functions added to a basis linear progression of the costs. It reads:

$$m_{(q)} = C_0\, q + \frac{C_1}{1 + e^{-\kappa(q-\tau_1)}} + \frac{C_2}{1 + e^{-\kappa(q-\tau_2)}}$$

where C_0 define the linear progression of the costs. The steps are located at thresholds τ_i with step heights C_i, for $i = 1, 2$. As for the disaster profile, the thresholds have to satisfy $0 < \tau_i \leq B$. Finally, κ defines the steepness of the steps. Sigmoid functions, commonly used in neural computing, enable the definition of arbitrarily sudden steps occurring without loss of mathematical continuity. They are used to simulate axon responses and subsequent decision processes.

Finally, the addressee's exposure is provided by $E_{(q)} = \mathcal{E}_{(q)}\, m_{(q)}$, as given in the main text. The corresponding curves are presented in Figure 5, Section 5.1.2.

12.4.2 EPS-Simulator

The methodology the simulator is based upon proceeds from a logical inversion. Weather events whose distribution follows a defined climatic profile are produced first, then the probabilistic forecasts, which are correlated to those events, are simulated. Indeed, instead of following the classical *ex ante* way peculiar to weather forecasting, an *ex post modus operandi* is applied. First should the event be generated, then the corresponding forecast that would have been produced by an Ensemble Prediction System will be simulated.

Precisely, the algorithm proceeds through the following steps:

Generate a meteorological event Starting from the climatic distribution defined in Section 5.1.1, generate an event of value $\tilde{Q}$. That event, represented by the downward pointing thick arrow in Figure 13, is a realisation of the random variable following the given climatic distribution, in this case a $\Gamma_{(r,s)}$ distribution with shape and scale parameters $r = 2, s = 3$.

Construct the corresponding probabilistic forecast In the simple approach chosen here, probabilistic forecasts follow normal distributions $N(\tilde{Q} + \mu, \sigma)$ whose departure μ to $\tilde{Q}$ and variance σ are themselves produced by random generators: μ follows another normal distribution $\mu = N_{(t,u)}$ with $t = 0, u = 2$ and σ follows another Γ distribution $\sigma = \Gamma_{(v,w)}$ with shape and scale parameters $v = 2, w = 2$.

Repeat the two steps as long as needed in order to produce a file $< event, forecast >$ from which statistics will be established.

The whole system is simply formed by a cascade of three levels of random generators. It is worth repeating that the probability of occurrence applied to the simulated forecast corresponds to climatic exposure the addressee is confronted with: the simulated forecasting model experiences the same climate as the addressee. Another point is that neither a temporal reference nor a forecast term is considered. However, a fading in the forecasting skill of the model can be simulated in modifying and/or increasing the parameters t, u, v, w introduced above. As a matter of fact, together with the two random generators $N_{(t,u)}$ and $\Gamma_{(v,w)}$, these four parameters govern the proprieties of the simulated model.

Three simulations are presented in Figure 35. The meteorological threshold is arbitrarily set to $Q = 10$. Meteorological events $\tilde{Q}$ are sketched by the vertical blue bars, the corresponding simulated EPS-forecasts by the yellow distributions. No warnings should be issued when blue bars fall into the green domain, up to Q, but ought to be in the pink domain. Probability of occurrence is measured in terms of the surface in pink domain below the yellow distribution and reported in black % on the panels. Were for example a probability threshold set at $P = 0.6$, then the first panel would represent a successfully warned event, the second one a correctly rejected non-event and the third one a missed event. Depending on the value of the meteorological threshold, up to 10.000 such events have been simulated in order to elaborate satisfying issuer's ROC Curves.

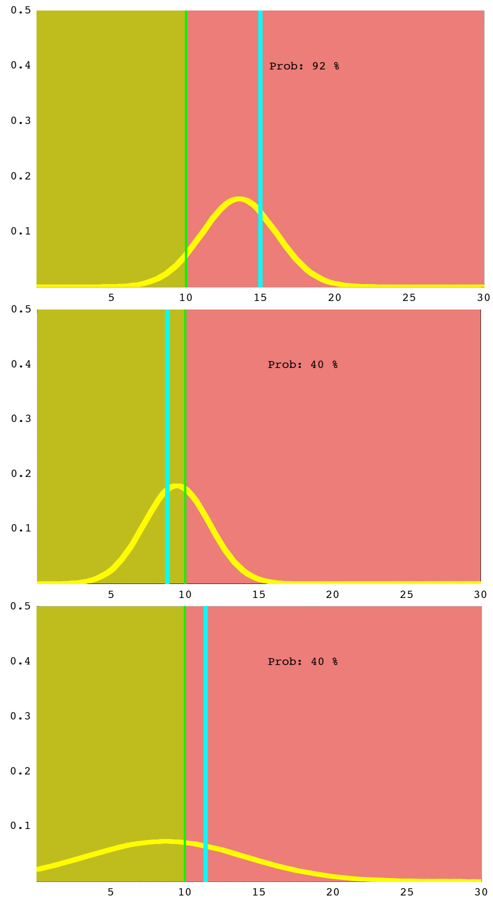

Figure 35: *Simulation of the EPS. The meteorological threshold is arbitrarily set to* $Q = 10$*. Meteorological events* $\tilde{Q}$ *are sketched by the vertical blue lines, simulated EPS-forecasts by the the yellow distributions.*

12.5 Derivation of $M_{R(H,F)}$ and $M_{A(H,Far)}$

The addressee's economic profile can be expressed either in terms of hit rate and false alarm rate, or of hit rate and false alarm ratio. Both derivations are presented hereafter with a, b, c and d defined in Section 12.1. The frequency of occurrence of meteorological events $\Omega = \frac{b+d}{a+b+c+d}$. The hit rate is given for both cases by $H = \frac{d}{b+d}$, the false alarm rate by $F = \frac{c}{a+c}$ for $M_{R(H,F)}$, the false alarm ratio by $Far = \frac{c}{c+d}$ for $M_{A(H,Far)}$.

12.5.1 Derivation of $M_{R(H,F)}$

The average costs the addressee is faced with during a period long enough to be of climatological relevance are expressed by equation (7) in Section 5.3. Making use of some elementary algebra, one reads (with "F" meaning "false alarm rate"):

$$
\begin{aligned}
M_R &= \frac{1}{a+b+c+d}[bL + C\,c + (C+\lambda)d] \\
&= \frac{1}{a+b+c+d}[Cc + d(C+\lambda-L) + (b+d)L] \\
&= C\frac{c}{a+c}\cdot\frac{a+c}{a+b+c+d} + \frac{d}{b+d}\cdot\frac{b+d}{a+b+c+d}(C+\lambda-L) \\
&\quad + \frac{b+d}{a+b+c+d}\,L \\
&= C\,F\frac{a+b+c+d-b-d}{a+b+c+d} + H\,\Omega\,(C+\lambda-L) + \Omega\,L \\
&= C\,F\,(1-\Omega) + H\,\Omega\,(C+\lambda-L) + \Omega\,L \\
&= L\,[\Gamma\,F\,(1-\Omega) + H\,\Omega\,(\Gamma+\Lambda-1) + \Omega].
\end{aligned}
$$

Finally:

$$M_{R(H,F)} = L\,[\Gamma\,F\,(1-\Omega) + H\,\Omega\,(\Gamma+\Lambda-1) + \Omega].$$

12.5.2 Derivation of $M_{A(H,Far)}$

The average costs the addressee is faced with during a period long enough to be of climatological relevance are expressed by equations (7) or (12) in Sections 5.3 and 7.1. Making use of the substitution *ad infinitum* of $c = Far\ (c+d)$ into itself (with "Far" meaning "false alarm ratio"), one reads:

$$
\begin{aligned}
M_A &= \frac{1}{a+b+c+d}\,[bL + Cc + (C+\lambda)d] \\
&= \frac{\Omega}{b+d}\,[bL + d\lambda + C(c+d)] \\
&= \Omega\,[\frac{b+d-d}{b+d}L + \frac{d}{b+d}\lambda + C\frac{c+d}{b+d}] \\
&= \Omega\,[(1-H)L + H\lambda + C\frac{Far\ (c+d)+d}{b+d}] \\
&= \Omega\,[(1-H)L + H\lambda + C(H + Far\ \frac{c+d}{b+d})] \\
&= \Omega\,[(1-H)L + H\lambda + C(H + Far\ \frac{Far\ (c+d)+d}{b+d})] \\
&= \Omega\,[(1-H)L + H\lambda + C(H + HFar + Far^2\ \frac{c+d}{b+d})] \\
&= \Omega\,[(1-H)L + H\lambda + C(H + HFar + HFar^2 + Far^3\ \frac{c+d}{b+d})] \\
&= \lim_{n\to\infty}\Omega\,[(1-H)L + H\lambda + C(H + HFar + HFar^2 + \cdots + Far^n\ \frac{c+d}{b+d})] \\
&= \Omega\,[(1-H)L + H\lambda + \frac{CH}{1-Far}].
\end{aligned}
$$

The geometric sequence (with $\eta = \frac{c+d}{b+d}$) is easily identified:

$$S_n = H + H\ Far + H\ Far^2 + H\ Far^3 + \cdots + H\ Far^{n-1} + \eta\ Far^n.$$

It satisfies:

$$S_n\ (1-Far) = H + (\eta - H)\ Far^n - \eta\ Far^{n+1}$$

and, therefore:

$$
\begin{aligned}
\lim_{n\to\infty} S_n &= \lim_{n\to\infty}\frac{1}{1-Far}(H + (\eta-H)\ Far^n - \eta\ Far^{n+1}) \\
&= \frac{H}{1-Far}; \qquad with\ Far < 1.
\end{aligned}
$$

The convergence of the infinite sequence occurs only for $Far < 1$, a requirement which can be expected to hold for the false alarm ratio. Indeed, this condition will be anchored in the setting of the application of M_A. The expression can be rearranged in a scalar product as follow:

$$\begin{aligned} M_{A(H,Far)} &= \Omega\,[(1-H)\,L + H\,\lambda + \frac{C\,H}{1-Far}] \\ &= L\,\Omega\,[(1-H) + H\,\frac{\lambda}{L} + \frac{C}{L}\,\frac{H}{1-Far}] \\ &= L\,\Omega\,[(1-H) + H\,\Lambda + \Gamma\,\frac{H}{1-Far}] \\ &= L\,\Omega \cdot \begin{bmatrix} 1 & \Lambda & \Gamma \end{bmatrix} \cdot \begin{bmatrix} 1-H \\ H \\ H\,(1-Far)^{-1} \end{bmatrix}. \end{aligned}$$

Once again do the climate burden $L\,\Omega$, the cost-loss and residual-loss ratios Γ and Λ nicely emerge. Together with the scalar product structure, they allow a cogent interpretation of the latter expression, presented in Section 7.1.

12.6 Determination of the optimal probability threshold P^*

As in Section 12.3, the derivation is carried through as a optimisation problem. However, the relationship between addressee's and issuer's profiles is now considered. The addressee's profile at meteorological threshold Q^* represents the utility function to be minimised under the constraint provided by the issuer's profile, expressed as his relative operating characteristic. Both sides of the following expression have therefore to be evaluated:

$$\frac{\partial_P H}{\partial_P Far}|_{Issuer_{(Q^*)}} = \frac{\partial H}{\partial Far}|_{Addressee_{(Q^*)}}$$

Issuer's profile For a given meteorological threshold Q^*, hit rates and false alarm ratios are extracted from the data emanating from the simulation for a sequence of probability thresholds spanning from $P = 0.05$ up to $P = 0.95$ in steps of 0.05. Polynomial fitting of that information delivers the values of the coefficients α_i and β_i of the polynomial expressions (of degree $N = 4$) given below.

$$H_{(p)} = \sum_{i=0}^{N} \alpha_i p^i \;;\; Far_{(p)} = \sum_{i=0}^{N} \beta_i p^i.$$

The smoothed orange ROC-curves presented throughout the document have been elaborated with these polynomials for $p \in [0.05, 0.95]$. Such computations have to be performed for each value of the meteorological threshold.

Finally, the determination of the required quotient is obvious:

$$\frac{\partial_P H}{\partial_P Far}|_{Issuer_{(Q^*)}} = (\sum_{i=1}^{N} i\alpha_i p^{i-1})(\sum_{i=1}^{N} i\beta_i p^{i-1})^{-1}.$$

Of course, some numerical checks have to be implemented in the simulations in order the prevent their blow-up in case of zero denominator.

Addressee's profile Considering that the derivation is to be realised for false alarm ratios lying below the limit induced by the climate burden (main text, Section 7.1), the expression (14) for $M_{A(H,Far)}$ given in Section 12.5.2 is preferred to the full-fledged formulation (15) of $\mathcal{M}_{Q^*(H,Far)\mathcal{S}}$. Taking into account the continuity of $M_{A(H,Far)}$ with respect to its variables H and Far, the theorem of the implicit function is applied in the sense that the iso-costs of $M_{A(H,Far)}$ are its implicit functions. On any iso-cost of monetary value $M_\$$ the relation $M_{A(H,Far)} = M_\$$ holds for the corresponding H and Far values. Thus, if H is the implicit function expressed in terms of Far, the total derivative is:

$$D_{Far} M_A(H_{(Far)}, Far) = \frac{\partial M_A}{\partial H}\frac{\partial H}{\partial Far} + \frac{\partial M_A}{\partial Far} = 0.$$

After few algebraic manipulations, the result reads:

$$\begin{aligned}\frac{\partial H}{\partial Far}|_{Addressee} &= -\frac{\partial M_A}{\partial Far}(\frac{\partial M_A}{\partial H})^{-1}\\ &= -\frac{\Gamma\, H}{1-Far}\cdot\frac{1}{\Gamma+(1-Far)(\Lambda-1)}\\ &= \frac{E_{(Q^*)}\, H}{(Far-1)(E_{(Q^*)}+Far-1)}.\end{aligned}$$

According to the definition of a rational addressee, the cost-loss ratio, having been substituted by its definition $\Gamma_{(Q^*)} = (1-\Lambda)\, E_{(Q^*)}$ in the last expression, the Λ ratio vanishes as well, leaving only the value of the disaster profile $E_{(Q^*)}$ in the equation.

Synthesis Requiring the identity of the slopes at the point on the ROC with curvilinear abscissa P and accordingly equating the two quantities:

$$\frac{\partial_P H}{\partial_P Far}|_{Issuer_{(Q^*)}} = \frac{\partial H}{\partial Far}|_{Addressee_{(Q^*)}}$$

leads to:

$$(\sum_{i=1}^{N} i\alpha_i p^{i-1})(\sum_{i=1}^{N} i\beta_i p^{i-1})^{-1} = \frac{E_{(Q^*)}\, H}{(Far-1)(E_{(Q^*)}+Far-1)}.$$

Substituing H and Far by their polynomial expressions on the right hand side yields:

$$(\sum_{i=1}^{N} i\alpha_i p^{i-1})(\sum_{i=1}^{N} i\beta_i p^{i-1})^{-1} = \frac{E_{(Q^*)} \sum_{i=0}^{N} \alpha_i p^i}{(\sum_{i=0}^{N} \beta_i p^i - 1)(E_{(Q^*)} + \sum_{i=0}^{N} \beta_i p^i - 1)}.$$

Finally, rearranging the terms provides the following polynomial equation $\mathcal{P}_{Q^*(p)} = 0$ in unknown p (of degree 11 for $N = 4$):

$$\begin{aligned} \mathcal{P}_{Q^*(p)} &= \sum_{i=1}^{N} i\alpha_i p^{i-1}(\sum_{i=0}^{N} \beta_i p^i - 1)(E_{(Q^*)} + \sum_{i=0}^{N} \beta_i p^i - 1) \\ &- E_{(Q^*)} \sum_{i=1}^{N} i\beta_i p^{i-1} \sum_{i=0}^{N} \alpha_i p^i \\ &= 0 \end{aligned}$$

whose only real root is the probability threshold P^* corresponding to the meteorological threshold Q^*. The location of the green dots pictured in all figures from Figure 19 onward, computed from the solution P^*, is given by the co-ordinates $\{H_{(P^*)}, Far_{(P^*)}\}$.

Remarks 1) This polynomial equation directly connects the forecasting skill of the issuer to the disaster profile of the addressee. All economic parameters, i.e. costs, losses, residual losses, as well as the deduced parameters Γ and Λ, after having served as scaffoldings in the elaboration of the relationship between both actors, disappear in this definitive formulation.

2) The differential equation can be integrated for itself, eventually yielding the expression of the iso-costs in the $\{H, Far\}$ frame. Starting with:

$$\frac{\partial H}{\partial Far} = \frac{E_{(Q^*)} H}{(Far - 1)(E_{(Q^*)} + Far - 1)}$$

one has:

$$H_{(Far)} = C \frac{Far - 1}{E_{(Q^*)} + Far - 1}.$$

This expression has been used to draw the green addressee's profiles in all figures from Figure 19 onward. C, the integration constant, is determined with the classical technique of the "variation of the constants", implemented through the Mathematica operator **FindMinimum**[...]. It is worth noticing that $Far < 1 - E_{(Q^*)}$. This condition, instrumental in the definition of the efficiency of the warning system, is discussed in Sections 9.4 and 12.7.

12.7 Backward derivation of the addressee's exposure

Assuming now that a numerical estimation $\frac{\Delta H}{\Delta Far}$ of the differential ratio $\frac{\partial H}{\partial Far}$ is known, an interesting relationship between both actor's profiles can be derived. The exposure $E_{(Q^*)}$ is isolated first:

$$E_{(Q^*)} = \frac{(Far-1)^2}{H\frac{\Delta Far}{\Delta H} - Far + 1}$$

then, using the definitions of Γ and Λ, and equation (9), one gets:

$$\frac{C}{L-\lambda} = \frac{\Gamma}{1-\Lambda} = E_{(Q^*)} = \frac{(Far-1)^2}{H\frac{\Delta Far}{\Delta H} - Far + 1}$$

12.7.1 Addressee's exposure

A deeper insight in this matter is attained with the introduction of the $u_{(p)}$ and $v_{(p)}$ reliability curves and the corresponding hit rate and false alarm ratios defined in Section 6.4, equations (10) and (11):

$$H_{(p)} = \frac{\int_p^1 u_{(\pi)} d\pi}{\int_0^1 u_{(\pi)} d\pi}; \qquad Far_{(p)} = \frac{\int_p^1 v_{(\pi)} d\pi}{\int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi}.$$

With these elements in hand, the relationship between the addressee's exposure and the reliability curves $u_{(p)}$ and $v_{(p)}$ at meteorological threshold Q^* and probability threshold p is simply given by:

$$E_{(Q^*)}|_p = \frac{u_{(p)}}{u_{(p)} + v_{(p)}}.$$

Proof: let us first algebraically express the differential equation given at the end of Section 12.6 in terms of $E_{(Q^*)}$:

$$\begin{aligned} E_{(Q^*)} &= \frac{(Far-1)^2}{H\frac{\partial Far}{\partial H} - Far + 1} \\ &= \frac{(Far-1)^2}{(\frac{1}{H}\frac{\partial H}{\partial p})^{-1} \cdot \frac{\partial Far}{\partial p} - Far + 1}. \end{aligned}$$

Let us now compute its components, firstly the complement to one of the false alarm ratio at a probability threshold p:

$$(Far-1)|_p = \frac{\int_p^1 v_{(\pi)} d\pi - \int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi}{\int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi} = -\frac{\int_p^1 u_{(\pi)} d\pi}{\int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi},$$

then le log-derivative of the hit rate with respect to the probability threshold p:

$$(\frac{1}{H}\frac{\partial H}{\partial p})|_p = \frac{\int_0^1 u_{(\pi)} d\pi}{\int_p^1 u_{(\pi)} d\pi} \cdot \frac{-u_{(p)}}{\int_0^1 u_{(\pi)} d\pi} = -\frac{u_{(p)}}{\int_p^1 u_{(\pi)} d\pi},$$

then the derivative of the false alarm ratio with respect to the probability threshold p:

$$\begin{aligned}\frac{\partial Far}{\partial p}|_p &= \frac{-v_{(p)} \int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi + (u_{(p)} + v_{(p)}) \int_p^1 v_{(\pi)} d\pi}{(\int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi)^2} \\ &= \frac{\int_p^1 [-u_{(\pi)} v_{(p)} - v_{(\pi)} v_{(p)} + u_{(p)} v_{(\pi)} + v_{(p)} v_{(\pi)}] d\pi}{(\int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi)^2} \\ &= \frac{u_{(p)} \int_p^1 v_{(\pi)} d\pi - v_{(p)} \int_p^1 u_{(\pi)} d\pi}{(\int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi)^2}.\end{aligned}$$

Finally, introducing those partial results into the formulation of the exposure $E_{(Q^*)}$ for the given meteorological and probability thresholds Q^* and p, one gets:

$$\begin{aligned}E_{(Q^*)}|_p &= \frac{(\int_p^1 u_{(\pi)} d\pi)^2}{(\int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi)^2 [-\frac{\int_p^1 u_{(\pi)} d\pi}{u_{(p)}} \cdot \frac{u_{(p)} \int_p^1 v_{(\pi)} d\pi - v_{(p)} \int_p^1 u_{(\pi)} d\pi}{(\int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi)^2} + \frac{\int_p^1 u_{(\pi)} d\pi}{\int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi}]} \\ &= \frac{\int_p^1 u_{(\pi)} d\pi}{(\int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi)^2 [-\frac{1}{u_{(p)}} \cdot \frac{u_{(p)} \int_p^1 v_{(\pi)} d\pi - v_{(p)} \int_p^1 u_{(\pi)} d\pi}{(\int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi)^2} + \frac{1}{\int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi}]} \\ &= \frac{\int_p^1 u_{(\pi)} d\pi}{-\int_p^1 v_{(\pi)} d\pi + \frac{v_{(p)}}{u_{(p)}} \int_p^1 u_{(\pi)} d\pi + \int_p^1 (u_{(\pi)} + v_{(\pi)}) d\pi} \\ &= \frac{\int_p^1 u_{(\pi)} d\pi}{\frac{v_{(p)}}{u_{(p)}} \int_p^1 u_{(\pi)} d\pi + \int_p^1 u_{(\pi)} d\pi} \\ &= \frac{1}{1 + \frac{v_{(p)}}{u_{(p)}}} \\ &= \frac{u_{(p)}}{u_{(p)} + v_{(p)}}.\end{aligned}$$

12.7.2 Sigmoidal interpretation

This interpretation is clued in the penultimate line of the preceding proof: the expression $(1 + \frac{v_{(p)}}{u_{(p)}})^{-1}$ conceals a sigmoidal structure. Indeed, considering the shape of the two functions u_p and $v_{(p)}$ introduced in Section 6.4 and sketched in Figure 15, a fair gamble consists in expressing them as exponential functions of the form $u_{(p)} = A\ e^{\alpha(p-\pi_\alpha)}$ and $v_{(p)} = B\ e^{\beta(p-\pi_\beta)}$ rather than as polynomials. Endorsing this assumption, the first expression in this section happens to be a sigmoid function (where $\alpha > \beta$):

$$\sigma_{(p)} = \frac{1}{1 + \frac{B}{A}\ e^{(\alpha\pi_\alpha - \beta\pi_\beta)}\ e^{p(\beta-\alpha)}}.$$

The parameters $\mathcal{P} = \{A, \alpha, \pi_\alpha, B, \beta, \pi_\beta\}$ are numerically estimated in solving the minimization problem:

$$\min_{\mathcal{P}} \left[\sum_{i=1}^{n} \left[\frac{1}{1 + \frac{\tilde{v}_i}{\tilde{u}_i}} - \sigma_{(\frac{i}{n})}\right]^2 \right]$$

where the $[\tilde{u}_i]_{i=1}^n$ and $[\tilde{v}_i]_{i=1}^n$ sequences with $n = 100$, are the frequencies collected after a simulation of the warning system, and sketched as blue and red dots in Figure 16. The minimization is carried out with the Mathematica operator **FindMinimum**[...], that truly does miracles. Sigmoids curves drawn in Figure 22 and 25 have been computed this way.

12.8 Efficiency measure of a warning system

The efficiency measure proposed is based on two assumptions.

The first one is that the addressee chooses either to assume his climate burden $L\,\Omega$, thus never takes mitigating or protective actions, or he co-operates with a warning issuer. In this constellation, no contract being considered between the addressee and an insurance company, the parameter setting $\mathcal{S} = 1$ given in the definition (15) for $\mathcal{M}_{Q^*(H,Far)\mathcal{S}}$ remains valid. This first assumption provides us with the upper bound - $L\,\Omega$ - of the efficiency measure.

The second assumption, defining the bottom bound of the efficiency measure, identifies that bound with the performance provided by an hypothetical perfect warning system whose hit rate would be one and false alarm ratio zero. Accordingly, such a bottom bound would given by $\mathcal{M}_{Q^*(1,0)\mathcal{S}}$.

Then, in this set-up, the efficiency is given by the difference between the climate burden $L\,\Omega$ and the burden occurring when warnings are issued and mitigating actions taken at a given meteorological threshold, hit rate and false alarm ratio. This difference is given by $L\,\Omega - \mathcal{M}_{Q^*(H,Far)\mathcal{S}}$ (> 0).

Considering that $\mathcal{M}_{Q^*(H,Far)\mathcal{S}} \in [\mathcal{M}_{Q^*(1,0)\mathcal{S}},\ L\,\Omega]$, the efficiency measure is normalized between zero and one, and reads:

$$\mathcal{F}_{Q^*(H,Far)\mathcal{S}} = \frac{L\,\Omega - \mathcal{M}_{Q^*(H,Far)\mathcal{S}}}{L\,\Omega - \mathcal{M}_{Q^*(1,0)\mathcal{S}}}$$

Taking finally into account the definition (15) of $\mathcal{M}_{Q^*(H,Far)}$ and the aforementioned definition of $\mathcal{F}_{Q^*(H,Far)}$, one notices that the previous expression can be simplified by the climate burden $L\,\Omega$, thus giving:

$$\mathcal{F}_{Q^*(H,Far)\mathcal{S}} = \frac{1 - Min_{Q^*(H,Far)\mathcal{S}}}{1 - Min_{Q^*(1,0)\mathcal{S}}}$$

The efficiency is zero when a warning system brings no positive departure in the addressee's financial burden from his climate burden. It is one when the warning system, working perfectly, detects all events and issues no false alarms. Following the observation already made in Section 7.4 concerning the disappearance of all financial parameters in the differential formulation of the addressee's profile, it is worth noticing that the climate burden $L\,\Omega$ vanishes here as it did by the derivation of equation (9) in Section 5.4.

Let us now simplify the above expression further, neglecting for this purpose the presence of an insurance company and assuming $F < 1 - E_{(Q)}$. Under such assumptions and the corresponding simplifications, the efficiency equation reads:

$$\mathcal{F}_{Q^*(H,Far)} = H \cdot \frac{1 - \Lambda - \frac{\Gamma}{1-Far}}{1 - \Lambda - \Gamma}$$

One notices that $\lim\limits_{Far \to 0} \mathcal{F}_{Q^*(H,Far)} = H$, respectively that $\mathcal{F}_{Q^*(H,Far)} = 0$ at $Far = 1 - E_{(Q^*)}$, a boundary already encountered in Section 12.6.

The first observation is useful. Taking into consideration the definition of the efficiency given in Section 9.2, equation (17), it appears that the iso-costs computed with the last expression given in Section 12.6 are iso-efficiency lines as well. Thus, the green addressee's profile intersects the vertical hit rate axis at abscissa zero and at ordinate value $H = \mathcal{F}_{Q^*(H,0)}$. By this way, the efficiency of the warning system is geometrically provided on the ROC diagram without any superfluous computation.

13 References

1. Richardson D.S. 2000. Skill and economic value of the ECMWF ensemble prediction system, (Q.J.R. Met. Soc. 2000; 126, pp. 649-667)

2. Ferrer Garcia and A. Sturzenegger F. 2001. Hedging Corporate Revenues with Weather Derivatives, a case study (University of Lausanne, HEC)

3. Jolliffe, I.I. and D.B. Stephenson, 2003. Forecast verification: A Practitioners Guide in Atmospheric Sciences (Wiley, pp. 66 - 76)

4. Nurni P. 2003. Recommendations on the Verification of local weather forecasts. (European Centre for Medium-Range Weather Forecasts; Technical Memorandum 430)

5. Grize L. Huss A. Thommen O. Schindler C. Braun-Fahrlaender C. Heatwave 2003 and mortality in Switzerland, 2005. (Swiss Med Wkly; 135, pp. 200 - 205)

6. Chavas J.P. 2004. Risk Analysis in Theory and Practice. (Elsevier Academic Press, Advanced Finance Series; pp. 31 - 51)

7. Marsigli C. Boccanera F. Montani A. Paccagnella T. 2005. The COSMO-LEPS mesoscale ensemble system: Validation of the methodology and verification. (Nonlinear processes in Geophysics; 12, pp. 527-536)

8. Theis. S. 2005. Deriving short-range forecasts from a deterministic high-resolution model. (Dissesrtation. Mathematisch- Naturwissenschafltichen Fakultaet der Rheinischen Friedrich-Wilhelms- Universitaet Bonn)

9. Germann U. Berenguer M. Sempere-Thores D. Salvade G. 2006. Ensemble radar precipitation estimation - a new topic on the radar horizon. (Proceedings 4th European Conf. on Radar in Meteorology and Hydrology; pp. 559 - 562)

10. Lhabitant F.S. 2004. Hedges Funds, Myths and Limits. (Wiley Finance)

11. Haechler P. 2003. Oral Communication. (Swiss Meteorological Society)

12. Brooks H.E. 2004. Bulletin of the American Meteorological Society 85 (6), pp. 837.

Author's Address

Jacques Ambühl
Bundesamt für Meteorologie und Klimatologie
Krähbühlstrasse 58
CH-8044 Zürich

jacques.ambuehl@meteoschweiz.ch
ambuhl@bluewin.ch

Veröffentlichungen der MeteoSchweiz

83 Ceppi, P: 2010, Spatial characteristics of gridded Swiss temperature trends: local and large-scale influences, 82 pp, CHF 76.-

82 Blanc, P: 2009, Ensemble-based uncertainty prediction for deterministic 2 m temperature forecasts, 90pp, CHF 78.-

81 Erdin R: 2009, Combining rain gauge and radar measurements of a heavy precipitation event over Switzerland: Comparison of geostatistical methods and investigation of important influencing factors, 109pp, CHF 81.-

80 Buzzi M: 2008, Challenges in Operational Numerical Weather Prediction at High Resolution in Complex Terrain, 186pp, CHF 103.-

79 Nowak D: 2008, Radiation and clouds: observations and model calculations for Payerne BSRN site, 101 pp, CHF 80.-

78 Arpagaus M, Rotach M, Ambrosetti P, Ament F, Appenzeller C, Bauer H-S, Bouttier F, Buzzi A, Corazza M, Davolio S, Denhard M, Dorninger M, Fontannaz L, Frick J, Fundel F, Germann U, Gorgas T, Grossi G, Hegg C, Hering A, Jaun S, Keil C, Liniger M, Marsigli C, McTaggart-Cowan R, Montani A, Mylne K, Ranzi R, Richard E, Rossa A, Santos-Muñoz D, Schär C, Seity Y, Staudinger M, Stoll M, Vogt S, Volkert H, Walser A, Wang Y, Werhahn J, Wulfmeyer V, Wunram C and Zappa M: 2009, MAP D-PHASE: Demonstrating forecast capabilities for flood events in the Alpine region. Report of the WWRP Forecast Demonstration Project D-PHASE submitted to the WWRP Scientific Steering Committee, 65pp, CHF 73.-

77 Rossa AM: 2007, MAP-NWS – an Optional EUMETNET Programme in Support of an Optimal Research Programme, 67pp, CHF 73.-

76 Baggenstos D: 2007, Probabilistic verification of operational monthly temperature forecasts, 52pp, CHF 69.-

75 Fikke S, Ronsten G, Heimo A, Kunz S, Ostrozlik M, Persson PE, Sabata J, Wareing B, Wichura B, Chum J, Laakso T, Säntti K and Makkonen L: 2007, COST 727: Atmospheric Icing on Structures Measurements and data collection on icing: State of the Art, 110pp, CHF 83.-

74 Schmutz C, Müller P und Barodte B: 2006, Potenzialabklärung für Public Private Partnership (PPP) bei MeteoSchweiz und armasuisse Immobilien, 82pp, CHF 76.-

73 Scherrer SC: 2006, Interannual climate variability in the European and Alpine region, 132pp, CHF 86.-

72 Mathis H: 2005, Impact of Realistic Greenhouse Gas Forcing on Seasonal Forecast Performance, 80pp, CHF 75.-

71 Leuenberger D: 2005, High-Resolution Radar Rainfall Assimilation: Exploratory Studies with Latent Heat Nudging, 103pp, CHF 81.-

70 Müller G und Viatte P: 2005, The Swiss Contribution to the Global Atmosphere Watch Programme – Achievements of the First Decade and Future Prospects, 112pp, CHF 83.-

69 Müller WA: 2004, Analysis and Prediction of the European Winter Climate, 115pp, CHF 34.

68 Bader S: 2004, Das Schweizer Klima im Trend: Temperatur- und Niederschlagsentwicklung seit 1864, 48pp, CHF 18.-

67 Begert M, Seiz G, Schlegel T, Musa M, Baudraz G und Moesch M: 2003, Homogenisierung von Klimamessreihen der Schweiz und Bestimmung der Normwerte 1961-1990, Schlussbericht des Projektes NORM90, 170pp, CHF 40.-

66 Schär C, Binder P and Richner H (Eds.): 2003, International Conference on Alpine Meteorology and MAP Meeting 2003, Extended Abstracts volumes A and B, 580pp, CHF 100.

Frühere Veröffentlichungen und Arbeitsberichte finden sich unter
www.meteoschweiz.ch » Forschung » Publikationen

Arbeitsberichte der MeteoSchweiz

229 Philipona R, Levrat G, Romanens G, Jeannet P, Ruffieux D and Calpini B, 2009: Transition from VIZ / Sippicanto ROTRONIC - A new humidity sensor for the SWISS SRS 400 Radiosonde, 37pp, CHF 66.-

228 MeteoSchweiz: 2009, Klimabericht Kanton Graubünden, 40pp, nur als .pdf erhältlich

227 MeteoSchweiz, 2009, Basisanalysen ausgewählter klimatologischer Parameter am Standort KKWLeibstadt, 135pp, CHF 88.-

226 MeteoSchweiz, 2009, Basisanalysen ausgewählter klimatologischer Parameter am Standort KKW Mühleberg, 136pp, CHF 88.-

225 MeteoSchweiz, 2009, Basisanalysen ausgewählter klimatologischer Parameter am Standort KKW Gösgen, 136 pp, CHF 88.-

224 MeteoSchweiz, 2009, Basisanalysen ausgewählter klimatologischer Parameter am Standort KKW Beznau, 135pp, CHF 88.-

223 Dürr B: 2008, Automatisiertes Verfahren zur Bestimmung von Föhn in den Alpentälern, 22pp, CHF 62.-

222 Schmutz C, Arpagaus M, Clementi L, Frei C, Fukutome S, Germann U, Liniger M und Schacher F: 2008, Meteorologische Ereignisanalyse des Hochwassers 8. bis 9. August 2007, 29pp, CHF 64.-

221 Frei C, Germann U, Fukutome S und Liniger M: 2008, Möglichkeiten und Grenzen der Niederschlagsanalysen zum Hochwasser 2005, 19pp, CHF 62.-

220 Ambühl J: 2008, Optimization of Warning Systems based on Economic Criteria, 79pp, CHF 75.-

219 Ceppi P, Della-Marta PM and Appenzeller C: 2008, Extreme Value Analysis of Wind Observations over Switzerland, 43pp, CHF 67.-

218 MeteoSchweiz (Hrsg): 2008, Klimaszenarien für die Schweiz – Ein Statusbericht, 50pp, CHF 69.-

217 Begert M: 2008, Die Repräsentativität der Stationen im Swiss National Basic Climatological Network (Swiss NBCN), 40pp, CHF 66.-

216 Della-Marta PM, Mathis H, Frei C, Liniger MA and Appenzeller C: 2007, Extreme wind storms over Europe: Statistical Analyses of ERA-40, 80pp, CHF 75.-

215 Begert M, Seiz G, Foppa N, Schlegel T, Appenzeller C und Müller G: 2007, Die Überführung der klimatologischen Referenzstationen der Schweiz in das Swiss National Climatological Network (Swiss NBCN), 47pp, CHF 68.-

214 Schmucki D und Weigel A: 2006, Saisonale Vorhersage in Tradition und Moderne: Vergleich der "Sommerprognose" des Zürcher Bööggs mit einem dynamischen Klimamodell, 46pp, CHF 68.-

213 Frei C: 2006, Eine Länder übergreifende Niederschlagsanalyse zum August Hochwasser 2005. Ergänzung zu Arbeitsbericht 211, 10pp, CHF 59.-

212 Z'graggen, L: 2006, Die Maximaltemperaturen im Hitzesommer 2003 und Vergleich zu früheren Extremtemperaturen, 74pp, CHF 75.-

211 MeteoSchweiz: 2006, Starkniederschlagsereignis August 2005, 63pp, CHF 72.-

210 Buss S, Jäger E and Schmutz C: 2005: Evaluation of turbulence forecasts with the aLMo, 58pp, CHF 70.–

209 Schmutz C, Schmuki D, Duding O, Rohling S: 2004, Aeronautical Climatological Information Sion LSGS, 77pp, CHF 25.–

Frühere Veröffentlichungen und Arbeitsberichte finden sich unter
www.meteoschweiz.ch » Forschung » Publikationen